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Cliquet option pricing with Meixner processes

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Abstract We investigate the pricing of cliquet options in a geometric Meixner model. The considered option is of monthly sum cap style while the underlying stock price model is driven by a pure-jump Meixner–Lévy process yielding Meixner distributed log-returns. In this setting, we infer semi-analytic expressions for the cliquet option price by using the probability distribution function of the driving Meixner–Lévy process and by an application of Fourier transform techniques. In an introductory section, we compile various facts on the Meixner distribution and the related class of Meixner–Lévy processes. We also propose a customized measure change preserving the Meixner distribution of any Meixner process.

Keywords Cliquet option pricing, path-dependent exotic option, equity indexed annuity, log-return of financial asset, Meixner distribution, Meixner–Lévy process, stochastic differential equation, probability measure change, characteristic function, Fourier transform

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1 Introduction

Cliquet option based contracts constitute a customized subclass of equity indexed annuities. The underlying options commonly are of monthly sum cap style paying a credited yield based on the sum of monthly-capped rates associated with some reference stock index. In this regard, cliquet type investments belong to the class of path-dependent exotic options. In [15] cliquet options are regarded as “the height of fashion in the world of equity derivatives”. In the literature, there are different pricing approaches for cliquet options involving e.g. partial differential equations (see [15]), Monte Carlo techniques (see [2]), numerical recursive algorithms related to inverse

Laplace transforms (see [9]) and analytical computation methods (see [3, 7, 8]). The present article belongs to the last category.

The aim of the present paper is to provide analytical pricing formulas for globally-floored locally-capped cliquet options with multiple resetting times where the underlying reference stock index is driven by a pure-jump time-homogeneous Meixner–Lévy process. In this setup, we derive cliquet option price formulas under two different approaches: once by using the distribution function of the driving Meixner–Lévy process and once by applying Fourier transform techniques (as proposed in [8]). All in all, the present article can be seen as an accompanying (but to a large degree self-contained) paper to [8], as it presents a specific application of the results derived in [8] to the class of Meixner–Lévy processes.

The paper is organized as follows: In Section 2 we compile facts on the Meixner distribution and the related class of stochastic Meixner–Lévy processes. In Section 3 we introduce a geometric pure-jump stock price model driven by a Meixner–Lévy process. In Section 3.1 we establish a customized structure preserving measure change from the risk-neutral to the physical probability measure. Section 4 is dedicated to the pricing of cliquet options. We obtain semi-analytic expressions for the cliquet option price by using the probability distribution function of the driving Meixner–Lévy process in Section 4.1 and by an application of Fourier transform techniques in Section 4.2. In Section 5 we draw the conclusions.

2 A review of Meixner processes

Let $(\Omega, \mathbb{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{Q})$ be a filtered probability space satisfying the usual hypotheses, i.e. $\mathcal{F}_t = \mathcal{F}_{t+} := \cap_{s>t}\mathcal{F}_s$ constitutes a right-continuous filtration and $\mathbb{F}$ denotes the sigma-algebra augmented by all $\mathbb{Q}$-null sets (cf. p. 3 in [10]). Here, $\mathbb{Q}$ is a risk-neutral probability measure and $0 < T < \infty$ denotes a finite time horizon. In the following, we compile various facts on the Meixner distribution and Meixner–Lévy processes from [1, 6, 12, 13] and [14].

A real-valued, càdlàg, pure-jump, time-homogeneous Lévy process $M = (M_t)_{t\in[0,T]}$ (with independent and stationary increments) satisfying $M_0 = 0$ is called Meixner (-Lévy) process with scaling parameter $\alpha > 0$, shape/skewness parameter $\beta \in (-\pi, \pi)$, peakedness parameter $\delta > 0$ and location parameter $\mu \in \mathbb{R}$, if M_t possesses the Lévy–Itô decomposition

$$M_t = \theta t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{Q}}(s, z) \tag{2.1}$$

where $\mathbb{R}_0 := \mathbb{R} \setminus \{0\}$, the drift parameter

$$\theta := \mu + \delta\alpha \tan(\beta/2) \tag{2.2}$$

is a real-valued constant and the $\mathbb{Q}$-compensated Poisson random measure (PRM) is given by

$$d\tilde{N}^{\mathbb{Q}}(s, z) := dN(s, z) - d\nu^{\mathbb{Q}}(z)ds \tag{2.3}$$

with positive and finite Meixner-type Lévy measure

$$d\nu^{\mathbb{Q}}(z) := \delta \frac{e^{\beta z/\alpha}}{z \sinh(\pi z/\alpha)} dz \tag{2.4}$$

(cf. [12, 13], Eq. (3) in [6]) satisfying $\nu^{\mathbb{Q}}(\{0\}) = 0$ and

$$\int_{\mathbb{R}_0} (1 \wedge z^2) d\nu^{\mathbb{Q}}(z) < \infty.$$

We denote the Lévy triplet of M_t by $(\theta, 0, \nu^{\mathbb{Q}})$. (Note that this notation is not entirely consistent with [6, 8].) We recall that M_t possesses moments of all orders (cf. Section 5.3.10 in [13]). Evidently, M_t has no Brownian motion part. Since

$$\int_{\mathbb{R}_0} |z| d\nu^{\mathbb{Q}}(z) = \infty$$

the process M_t possesses infinite variation (cf. Section 5.3.10 in [13]). We write for any fixed $t \in [0, T]$

$$M_t \sim \mathcal{M}(\alpha, \beta, \delta t, \mu t)$$

(cf. Section 3.6 in [1]) and say that M is Meixner distributed under $\mathbb{Q}$ with parameters α, β, δ and μ. From (2.1) and (2.2) we instantly receive the mean value

$$\mathbb{E}_{\mathbb{Q}}[M_t] = \theta t = \mu t + \delta t \alpha \tan(\beta/2) \tag{2.5}$$

standing in accordance with Eq. (11) in [6]. The variance, skewness and kurtosis of M_t are respectively given by

$$\mathbb{Var}_{\mathbb{Q}}[M_t] = \frac{\delta t}{2} \frac{\alpha^2}{\cos^2(\beta/2)}, \qquad \mathbb{S}_{\mathbb{Q}}[M_t] = \sqrt{2/(\delta t)} \sin(\beta/2),$$
$$\mathbb{K}_{\mathbb{Q}}[M_t] = 3 + \frac{2 - \cos(\beta)}{\delta t}$$

(cf. Table 6 in [1]). Furthermore, for all $x \in \mathbb{R}$ and $t \in [0, T]$ the real-valued probability density function (pdf) of M_t under $\mathbb{Q}$ reads as

$$f_{M_t}(x) := \frac{(2\cos(\beta/2))^{2\delta t}}{2\pi\alpha\Gamma(2\delta t)} e^{\beta(x-\mu t)/\alpha} \left| \Gamma\left(\delta t + i\frac{x - \mu t}{\alpha}\right) \right|^2 \tag{2.6}$$

(cf. [1, 12], Eq. (4) in [6]) wherein

$$\Gamma(\zeta) := \int_0^\infty u^{\zeta-1} e^{-u} du$$

denotes the gamma function which is defined for all $\zeta \in \mathbb{C}$ with $Re(\zeta) > 0$. Taking the definition of the gamma function and Euler's formula into account, we get

$$\Gamma\left(\delta t + i\frac{x - \mu t}{\alpha}\right) = \int_{0+}^\infty u^{\delta t - 1} e^{-u} \cos\left(\frac{x - \mu t}{\alpha} \ln u\right) du$$

$$+ i \int_{0+}^{\infty} u^{\delta t - 1} e^{-u} \sin\left(\frac{x - \mu t}{\alpha} \ln u\right) du$$

which implies

$$\left| \Gamma\left(\delta t + i \frac{x - \mu t}{\alpha}\right) \right|^2 = \left(\int_{0+}^{\infty} u^{\delta t - 1} e^{-u} \cos\left(\frac{x - \mu t}{\alpha} \ln u\right) du \right)^2 + \left(\int_{0+}^{\infty} u^{\delta t - 1} e^{-u} \sin\left(\frac{x - \mu t}{\alpha} \ln u\right) du \right)^2.$$

Note that the latter object appears in (2.6). The cumulative distribution function (cdf) of M_t does not possess a closed form representation but it can be computed numerically. Further on, the characteristic function of M_t can be computed by the Lévy–Khinchin formula (see e.g. [4, 5, 11, 13]) due to

$$\phi_{M_t}(u) := \mathbb{E}_{\mathbb{Q}}\left[e^{iuM_t}\right] = e^{\psi(u)t} \tag{2.7}$$

with $i^2 = -1$, $u \in \mathbb{R}$, $t \in [0, T]$ and a characteristic exponent

$$\psi(u) := iu\left[\mu + \delta\alpha \tan\left(\frac{\beta}{2}\right)\right] + \delta \int_{\mathbb{R}_0} \frac{e^{iuz} - 1 - iuz}{z} \frac{e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz. \tag{2.8}$$

Moreover, let us define the Fourier transform, respectively inverse Fourier transform, of a deterministic function $q \in \mathcal{L}^1(\mathbb{R})$ via

$$\hat{q}(y) := \int_{\mathbb{R}} q(x) e^{iyx} dx, \, q(x) = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{q}(y) e^{-iyx} dy.$$

Then for all $u \in \mathbb{R}$ and $t \in [0, T]$ we receive the well-known relationship

$$\phi_{M_t}(u) = \hat{f}_{M_t}(u)$$

where $\hat{f}_{M_t}$ denotes the Fourier transform of the density function f_{M_t} defined in (2.6). An application of the inverse Fourier transform yields

$$f_{M_t}(x) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{\psi(u)t - iux} du$$

thanks to (2.7). On the other hand, from Eq. (1) in [6] we know that

$$\phi_{M_t}(u) = e^{iu\mu t} \left(\frac{\cos(\beta/2)}{\cosh((\alpha u - i\beta)/2)} \right)^{2\delta t} \tag{2.9}$$

where $u \in \mathbb{R}$ and $t \in [0, T]$. Taking the logarithm in (2.7) and (2.9), we finally deduce

$$\psi(u) = iu\mu + 2\delta\left[\ln\cos\left(\frac{\beta}{2}\right) - \ln\cosh\left(\frac{\alpha u - i\beta}{2}\right)\right]. \tag{2.10}$$

Further on, for the Meixner distribution the following properties are well-known (cf. [14], Section 5.3.10 in [13], Section 3.6 in [1], Corollary 1 in [6]).

Lemma 2.1. *(a) If $X \sim \mathcal{M}(\alpha, \beta, \delta, \mu)$, then $cX + m \sim \mathcal{M}(c\alpha, \beta, \delta, c\mu + m)$ with constants $c > 0$ and $m \in \mathbb{R}$.*

(b) If $X_1 \sim \mathcal{M}(\alpha, \beta, \delta_1, \mu_1)$ and $X_2 \sim \mathcal{M}(\alpha, \beta, \delta_2, \mu_2)$ are independent random variables, then $X_1 + X_2 \sim \mathcal{M}(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)$.

(c) The characteristic function $\phi_X(u; \alpha, \beta, \delta, \mu)$ of a Meixner distributed random variable $X \sim \mathcal{M}(\alpha, \beta, \delta, \mu)$ satisfies

$$\phi_X(u; \alpha, \beta, \delta, \mu) = \phi_X(u; \alpha, \beta, \delta/n, \mu/n)^n$$

for arbitrary $n \in \mathbb{N}$ such that the Meixner distribution is infinitely divisible.

3 A stock price model driven by a Meixner process

Let $t \in [0, T]$ and define the stochastic stock price process S_t via

$$S_t := S_0 e^{M_t + bt} \tag{3.1}$$

with deterministic initial value S_0, a constant $b \in \mathbb{R}$ and a real-valued Meixner process

$$M_t = \theta t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{Q}}(s, z)$$

such as introduced in (2.1)–(2.4). Here, the constant b provides some additional degree of freedom which is introduced in order to ensure the arbitrage-freeness of the stock price model. More details on this topic will be given below. Verify that (3.1) belongs to the same model class (geometric Lévy models) as (2.2)–(2.3) in [8]. We next introduce the historical filtration

$$\mathcal{F}_t := \sigma\{S_u : 0 \le u \le t\} = \sigma\{M_u : 0 \le u \le t\}.$$

Using Itô's formula, we obtain the stochastic differential equation (SDE)

$$\frac{dS_t}{S_{t-}} = \left(\theta + b + \int_{\mathbb{R}_0} [e^z - 1 - z] d\nu^{\mathbb{Q}}(z)\right) dt + \int_{\mathbb{R}_0} [e^z - 1] d\tilde{N}^{\mathbb{Q}}(t, z)$$

under $\mathbb{Q}$. Let us further define the discounted stock price via

$$\hat{S}_t := \frac{S_t}{B_t}$$

where S_t is such as defined in (3.1) and $B_t := e^{rt}$ is the value of a bank account with normalized initial capital $B_0 = 1$ and risk-less interest rate $r > 0$. Due to (3.1) we find

$$\hat{S}_t = S_0 e^{M_t + (b-r)t}$$

while Itô's formula yields the following SDE under $\mathbb{Q}$

$$\frac{d\hat{S}_t}{\hat{S}_{t-}} = \left(\theta + b - r + \int_{\mathbb{R}_0} [e^z - 1 - z] d\nu^{\mathbb{Q}}(z)\right) dt + \int_{\mathbb{R}_0} [e^z - 1] d\tilde{N}^{\mathbb{Q}}(t, z).$$

In accordance to no-arbitrage theory, the discounted stock price $\hat{S}$ must form a martingale under the risk-neutral probability measure $\mathbb{Q}$. For this reason, we require the drift restriction

$$b = r - \theta - \int_{\mathbb{R}_0} \big[e^z - 1 - z\big] d\nu^{\mathbb{Q}}(z). \tag{3.2}$$

With this particular choice of the coefficient b, we deduce

$$\frac{dS_t}{S_{t-}} = r dt + \int_{\mathbb{R}_0} \big[e^z - 1\big] d\tilde{N}^{\mathbb{Q}}(t, z)$$

under $\mathbb{Q}$. Combining (3.1) and (3.2), we receive

$$S_t = S_0 e^{rt} \exp\left\{ \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{Q}}(s, z) - \int_0^t \int_{\mathbb{R}_0} \big[e^z - 1 - z\big] d\nu^{\mathbb{Q}}(z) ds \right\}$$

where the last factor on the right hand side constitutes a Doléans-Dade exponential which again shows the $\mathbb{Q}$-martingale property of the discounted stock price process $\hat{S}_t = S_t e^{-rt}$. Moreover, taking (2.2), (2.4), (2.8) and (2.10) into account, Eq. (3.2) can be expressed as

$$b = r - \psi(-i) = r - \mu - 2\delta \ln\left(\frac{\cos(\beta/2)}{\cos((\alpha + \beta)/2)} \right). \tag{3.3}$$

Unless otherwise stated, from now on we assume that the constant $b \in \mathbb{R}$ appearing in (3.1) is such as given in (3.3). Though constituting an admissible choice, taking $b = 0$ in (3.1)–(3.3) might be too restrictive in practical applications. In the following, we investigate the log-returns related to our model (3.1). For an arbitrary time step $\Delta > 0$ and $t \leq T - \Delta$ we obtain

$$\ln\left(\frac{S_{t+\Delta}}{S_t} \right) \cong M_\Delta + b\Delta \sim \mathcal{M}\big(\alpha, \beta, \delta\Delta, (\mu + b)\Delta\big)$$

by Lemma 2.1 (a). Here, the symbol $\cong$ denotes equality in distribution. Hence, in our stock price model (3.1) the log-returns are Meixner distributed. We stress that in [12] it was shown that the Meixner distribution fits empirical financial log-returns very well. Furthermore, for $n \in \mathbb{N}$ we introduce the time partition $\mathcal{P} := \{0 < t_0 < t_1 < \cdots < t_n \leq T\}$ and define the return/revenue process associated with the period $[t_{k-1}, t_k]$ via

$$R_k := \frac{S_{t_k} - S_{t_{k-1}}}{S_{t_{k-1}}}$$

where $k \in \{1, \ldots, n\}$. A substitution of (3.1) into the latter equation yields

$$R_k = e^{Y_{t_k} - Y_{t_{k-1}}} - 1 \tag{3.4}$$

where

$$Y_t := M_t + bt \sim \mathcal{M}\big(\alpha, \beta, \delta t, (\mu + b)t\big)$$

is a Meixner–Lévy process. Taking (2.1)–(2.4) and (3.3) into account, we get

$$Y_t = \gamma t + \int_0^t \int_{\mathbb{R}_0} z dN(s, z) \tag{3.5}$$

where

$$\gamma := r + \delta\left[\alpha \tan\left(\frac{\beta}{2}\right) - 2\ln\left(\frac{\cos(\beta/2)}{\cos((\alpha+\beta)/2)}\right) - \int_{\mathbb{R}_0} \frac{e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz\right]$$

is a real-valued constant. Recall that the Meixner–Lévy process Y given in (3.5) above just is a special case of the more general Lévy process X defined in Eq. (2.3) in [8]. For this reason, the cliquet option pricing results derived in [8] simultaneously apply in our current Meixner modeling case. More details on this topic are given in Section 4 below. Also note that $R_1, \ldots, R_n$ are $\mathbb{Q}$-independent random variables and that $R_k > -1$ $\mathbb{Q}$-almost surely for all k. Since Y is a Lévy process under $\mathbb{Q}$, we observe $Y_{t_k} - Y_{t_{k-1}} \cong Y_\tau$ (stationary increments) where $\tau := t_k - t_{k-1}$ (equidistant partition). Here, the symbol $\cong$ denotes equality in distribution. For the sake of notational simplicity, we always work under the assumption of equidistant time points in the following, unless otherwise stated. Taking (3.4) into account, we obtain the subsequent relationship between the cumulative distribution functions of R_k and Y_τ

$$\mathbb{Q}(R_k \leq \xi) = \mathbb{Q}\big(Y_\tau \leq \ln(1+\xi)\big) \tag{3.6}$$

where $\xi > -1$ is an arbitrary real-valued constant.

3.1 A structure preserving measure change to the physical probability measure

Recall that we worked under the risk-neutral probability measure $\mathbb{Q}$ in the previous sections. Since log-returns of financial assets are commonly observed under the physical measure $\mathbb{P}$ (instead of under $\mathbb{Q}$), we establish a measure change from $\mathbb{Q}$ to $\mathbb{P}$ in the sequel. In this context, we have to pay special attention to the so-called structure preserving property of the measure change, as the log-returns under $\mathbb{P}$ shall again follow a Meixner distribution. In other words, the Meixner process M_t introduced in (2.1) under $\mathbb{Q}$ shall also be a Meixner process under $\mathbb{P}$. First of all, for $t \in [0, T]$ we define the Radon–Nikodym density process

$$\Lambda_t := \frac{d\mathbb{P}}{d\mathbb{Q}}\bigg|_{\mathcal{F}_t} := \exp\left\{\int_0^t \int_{\mathbb{R}_0} h(z) d\tilde{N}^{\mathbb{Q}}(s,z) - \int_0^t \int_{\mathbb{R}_0} \big[e^{h(z)} - 1 - h(z)\big] d\nu^{\mathbb{Q}}(z) ds\right\}$$

where the $\mathbb{Q}$-compensated PRM $\tilde{N}^{\mathbb{Q}}$ and the corresponding Lévy measure $\nu^{\mathbb{Q}}$ are such as defined in (2.3), respectively (2.4), while $h(z)$ is a time-independent deterministic function on $\mathbb{R}_0$. Recall that we may write

$$\Lambda_t = \frac{e^{L_t}}{\mathbb{E}_{\mathbb{Q}}[e^{L_t}]}$$

with a local $\mathbb{Q}$-martingale process

$$L_t := \int_0^t \int_{\mathbb{R}_0} h(z) d\tilde{N}^{\mathbb{Q}}(s,z)$$

such that the density process Λ is detected to be of Esscher transform type. Note that Λ is a discontinuous Doléans-Dade exponential which constitutes a local martingale under $\mathbb{Q}$ satisfying the SDE

$$d\Lambda_t = \Lambda_{t-} \int_{\mathbb{R}_0} \big[e^{h(z)} - 1\big] d\tilde{N}^{\mathbb{Q}}(t, z).$$

In accordance to Theorem 12.21 in [5], we further impose the Novikov condition

$$\mathbb{E}_{\mathbb{Q}}\left[\exp\left\{\int_0^t \int_{\mathbb{R}_0} \big[1 - e^{h(z)} + h(z)e^{h(z)}\big] d\nu^{\mathbb{Q}}(z) ds\right\}\right] < \infty$$

for all $t \in [0, T]$. Then it holds $\mathbb{E}_{\mathbb{Q}}[\Lambda_t] \equiv 1$ for all $t \in [0, T]$ such that Λ constitutes a true $\mathbb{Q}$-martingale. Hence, we may apply Girsanov's theorem stating that

$$d\tilde{N}^{\mathbb{P}}(s, z) := dN(s, z) - d\nu^{\mathbb{P}}(z) ds \tag{3.7}$$

constitutes the $\mathbb{P}$-compensated Poisson random measure with Lévy measure

$$d\nu^{\mathbb{P}}(z) := e^{h(z)} d\nu^{\mathbb{Q}}(z). \tag{3.8}$$

Note that the Novikov condition is equivalent to requiring that

$$\int_{\mathbb{R}_0} \big[1 - e^{h(z)} + h(z)e^{h(z)}\big] d\nu^{\mathbb{Q}}(z) < \infty$$

since h and $\nu^{\mathbb{Q}}$ both are deterministic. A combination of (2.1), (2.3), (3.7) and (3.8) yields the following Lévy–Itô decomposition

$$M_t = \left(\theta + \int_{\mathbb{R}_0} z\big[e^{h(z)} - 1\big] d\nu^{\mathbb{Q}}(z)\right) t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{P}}(s, z) \tag{3.9}$$

under $\mathbb{P}$ where θ, $\nu^{\mathbb{Q}}$ and $\tilde{N}^{\mathbb{P}}$ are such as defined in (2.2), (2.4) and (3.7), respectively. The remaining challenge now consists in finding an appropriate function $h(z)$ which, firstly, fulfills the Novikov condition, secondly, guarantees that $\nu^{\mathbb{P}}$ in (3.8) constitutes a Lévy measure of Meixner-type and, thirdly, ensures that M_t in (3.9) is a Meixner–Lévy process. In this regard, we propose to work with the specification

$$h(z) := \frac{\beta^* - \beta}{\alpha} z \tag{3.10}$$

from now on. Herein, the constant skewness parameter $\beta^* \in (-\pi, \pi)$ satisfies $\beta^* \neq \beta$ while $\alpha > 0$ is the scaling parameter introduced above. Note that taking $\beta^* = \beta$ would imply $h(z) \equiv 0$ and hence, $\mathbb{P} = \mathbb{Q}$. Combining (3.8) with (2.4) and (3.10), we deduce

$$d\nu^{\mathbb{P}}(z) = \delta \frac{e^{\beta^* z/\alpha}}{z \sinh(\pi z/\alpha)} dz \tag{3.11}$$

which constitutes a Meixner-type Lévy measure with parameters α, β^* and δ [recall Eq. (2.4)]. Moreover, with respect to (3.8) and (3.10), we obtain

$$\int_{\mathbb{R}_0} \big[1 - e^{h(z)} + h(z)e^{h(z)}\big] d\nu^{\mathbb{Q}}(z) = \nu^{\mathbb{Q}}(\mathbb{R}_0) - \nu^{\mathbb{P}}(\mathbb{R}_0) + \frac{\beta^* - \beta}{\alpha} \int_{\mathbb{R}_0} z d\nu^{\mathbb{P}}(z)$$

which is finite, because the Lévy measures $\nu^{\mathbb{Q}}$ and $\nu^{\mathbb{P}}$ are finite. Thus, the function $h(z)$ defined in (3.10) indeed fulfills the Novikov condition. Further on, we take (2.2), (2.4), (3.9) and (3.10) into account and receive

$$\mathbb{E}_{\mathbb{P}}[M_t] = \theta^* t \tag{3.12}$$

with drift parameter

$$\theta^* := \mu^* + \delta\alpha \tan\left(\frac{\beta^*}{2}\right)$$

and a constant and real-valued location parameter

$$\mu^* := \mu + \delta\left[\alpha \tan\left(\frac{\beta}{2}\right) - \alpha \tan\left(\frac{\beta^*}{2}\right) + \int_{\mathbb{R}_0} \frac{e^{\beta^* z/\alpha} - e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz\right]. \tag{3.13}$$

Note in passing that (3.12) possesses the same structure as (2.5). Also verify that

$$\theta^* - \theta = \delta \int_{\mathbb{R}_0} \frac{e^{\beta^* z/\alpha} - e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz = \int_{\mathbb{R}_0} z\big[d\nu^{\mathbb{P}}(z) - d\nu^{\mathbb{Q}}(z)\big] \tag{3.14}$$

due to (2.2), (2.4), (3.11), (3.12) and (3.13). All in all, combining (3.9) with (2.4), (3.10) and (3.14), we conclude that

$$M_t = \theta^* t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{P}}(s, z) \tag{3.15}$$

which constitutes a Meixner–Lévy process under $\mathbb{P}$ with distribution

$$M_t \sim \mathcal{M}\big(\alpha, \beta^*, \delta t, \mu^* t\big). \tag{3.16}$$

For this reason, we call the recently introduced measure change *structure preserving*. Recall that in Section 2 under $\mathbb{Q}$ we observed $M_t \sim \mathcal{M}(\alpha, \beta, \delta t, \mu t)$ on the other hand. Thus, the proposed measure change does neither affect the scaling parameter α nor the peakedness parameter δ whereas both the skewness parameter β and the location parameter μ are changed. Moreover, the Lévy triplet of the process M claimed in (3.15) is given by $(\theta^*, 0, \nu^{\mathbb{P}})$. In analogy to the result provided in the sequel of (3.3), we remark that under $\mathbb{P}$ it holds

$$\ln\left(\frac{S_{t+\Delta}}{S_t}\right) \cong M_\Delta + b\Delta \sim \mathcal{M}\big(\alpha, \beta^*, \delta\Delta, \big(\mu^* + b\big)\Delta\big)$$

due to Lemma 2.1 (a). Here, $\Delta > 0$ is a constant and b is such as given in (3.3). Hence, if we specify the Radon–Nikodym function $h(z)$ like in (3.10), then the log-returns again are Meixner distributed under the real-world probability measure $\mathbb{P}$. Further note that

$$\theta t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{Q}}(s, z) = M_t = \theta^* t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{P}}(s, z)$$

holds $\mathbb{P}$- respectively $\mathbb{Q}$-almost surely for all $t \in [0, T]$ thanks to (2.1) and (3.15). We obtain the following expressions for the variance, skewness and kurtosis of M_t under $\mathbb{P}$

$$\mathbb{V}\mathrm{ar}_{\mathbb{P}}[M_t] = \frac{\delta t}{2} \frac{\alpha^2}{\cos^2(\beta^*/2)}, \qquad \mathbb{S}_{\mathbb{P}}[M_t] = \sqrt{2/(\delta t)} \sin\big(\beta^*/2\big),$$
$$\mathbb{K}_{\mathbb{P}}[M_t] = 3 + \frac{2 - \cos(\beta^*)}{\delta t}$$

while under $\mathbb{P}$ the density and characteristic function of M_t are such as given in (2.6) and (2.9) but with μ and β therein replaced by μ^* and β^*, respectively.

3.1.1 *A generalized structure preserving measure change*

In this section, we present a generalized structure preserving measure change from the risk-neutral to the physical probability measure. Recall that the measure change proposed above only affects the skewness parameter β and the location parameter μ whereas both the scaling parameter α and the peakedness parameter δ remain untouched. From a practical point of view, this fact might be regarded as an advantage, as there is no need to recalibrate the parameters α and δ when changing from the risk-neutral to the physical probability measure. Conversely, the described feature might likewise cause some difficulties when it comes to calibrating under $\mathbb{P}$, since the values of the parameters α and δ have to be the same as under $\mathbb{Q}$ yielding some loss of flexibility. To avoid this disadvantage, we now propose a generalized measure change which affects each of the four parameters of the Meixner distribution. For this purpose, we presently require that the Meixner–Lévy measure under $\mathbb{P}$ is of the form

$$d\nu^{\mathbb{P}}(z) = \delta^* \frac{e^{\beta^* z/\alpha^*}}{z \sinh(\pi z/\alpha^*)} dz \tag{3.17}$$

[cf. Equation (2.4)] with new parameters $\alpha^* > 0$, $\beta^* \in (-\pi, \pi)$ and $\delta^* > 0$ which are different from α, β and δ introduced previously under $\mathbb{Q}$. Following this approach, we are led to the equality

$$e^{h(z)} = \frac{\delta^* \sinh(\pi z/\alpha)}{\delta \sinh(\pi z/\alpha^*)} \exp\left\{\left(\frac{\beta^*}{\alpha^*} - \frac{\beta}{\alpha}\right) z\right\}$$

due to (3.17), (3.8) and (2.4). Taking the logarithm in the latter equation, we receive

$$h(z) = \left(\frac{\beta^*}{\alpha^*} - \frac{\beta}{\alpha}\right) z + \ln\left(\frac{\delta^* \sinh(\pi z/\alpha)}{\delta \sinh(\pi z/\alpha^*)}\right) \tag{3.18}$$

which corresponds to (3.10) above. Note that (3.18) is well-defined for all $z \in \mathbb{R}_0$ and that we obtain (3.10), if we take $\alpha^* = \alpha$ and $\delta^* = \delta$ in (3.18). Hence, (3.10) is a special case of (3.18). Moreover, if we take $\alpha^* = \alpha$ and $\beta^* = \beta$ in (3.18), then we get $h(z) = \ln \delta^* - \ln \delta$ which is constant and independent of z. We summarize our findings in the following proposition.

Proposition 3.1. *Consider the measure change from the risk-neutral to the physical probability measure with Radon–Nikodym density process Λ such as defined at the beginning of Section 3.1. Then the new Lévy measure $\nu^{\mathbb{P}}$ under $\mathbb{P}$ is of Meixner-type again, if and only if the Radon–Nikodym function $h(z)$ is of the form (3.18).*

In the sequel, we investigate the distributional properties of the corresponding Meixner–Lévy process under $\mathbb{P}$ related to the Radon–Nikodym function $h(z)$ given in (3.18). A substitution of (2.2), (2.4) and (3.18) into (3.9) yields the following Lévy–Itô decomposition under $\mathbb{P}$

$$M_t = \overline{\theta} t + \int_0^t \int_{\mathbb{R}_0} z d\tilde{N}^{\mathbb{P}}(s, z) \tag{3.19}$$

with deterministic and real-valued drift parameter

$$\overline{\theta} := \mu + \delta\alpha \tan\left(\frac{\beta}{2}\right) + \delta^* \int_{\mathbb{R}_0} \frac{e^{\beta^* z/\alpha^*}}{\sinh(\pi z/\alpha^*)} dz - \delta \int_{\mathbb{R}_0} \frac{e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz.$$

The Meixner–Lévy process M_t given in (3.19) possesses the Lévy triplet $(\overline{\theta}, 0, \nu^{\mathbb{P}})$ where $\nu^{\mathbb{P}}$ is such as claimed in (3.17). In the next step, we require that $\overline{\theta}$ is of the form (2.2), i.e.

$$\overline{\theta} = \overline{\mu} + \delta^* \alpha^* \tan\left(\beta^*/2\right)$$

with some new location parameter $\overline{\mu} \in \mathbb{R}$. Following this onset, we deduce

$$\overline{\mu} = \mu + \delta\alpha \tan\left(\frac{\beta}{2}\right) - \delta^*\alpha^* \tan\left(\frac{\beta^*}{2}\right) + \delta^* \int_{\mathbb{R}_0} \frac{e^{\beta^* z/\alpha^*}}{\sinh(\pi z/\alpha^*)} dz - \delta \int_{\mathbb{R}_0} \frac{e^{\beta z/\alpha}}{\sinh(\pi z/\alpha)} dz.$$

Hence, if the measure change from $\mathbb{Q}$ to $\mathbb{P}$ is performed with the Radon–Nikodym function $h(z)$ defined in (3.18), then the corresponding Meixner–Lévy process M_t again is Meixner distributed under $\mathbb{P}$ with parameters

$$M_t \sim \mathcal{M}\left(\alpha^*, \beta^*, \delta^* t, \overline{\mu} t\right). \tag{3.20}$$

If we compare (3.20) with (3.16), we see that in the generalized measure change related to (3.18) each of the four parameters of the Meixner–Lévy process M is affected.

4 Cliquet option pricing in a geometric Meixner model

This section is devoted to the pricing of cliquet options in the Meixner stock price model presented in Chapter 3. Since the Meixner process Y in (3.5) above just is a special case of the more general Lévy process X defined in Eq. (2.3) in [8], the cliquet option pricing results derived in [8] simultaneously apply to our present Meixner–Lévy modeling case. The details are worked out in the remainder of the current section. Parallel to [8] and Eq. (1.1) in [3], we consider a monthly sum cap style cliquet option with payoff

$$H_T = K + K \max\left\{g, \sum_{k=1}^{n} \min\{c, R_k\}\right\}$$

where T is the maturity time, K denotes the notional (i.e. the initial investment), g is the guaranteed rate at maturity, $c \geq 0$ is the local cap and R_k is the return process given in (3.4). Recall that the payoff H_T is globally-floored by the constant $K(1+g)$ and locally-capped by c. By a case distinction, we get

$$H_T = K \max\left\{1+g, 1+\sum_{k=1}^{n} \min\{c, R_k\}\right\} = K\left(1+g+\max\left\{0, \sum_{k=1}^{n} Z_k\right\}\right)$$

where for all $k \in \{1, \ldots, n\}$ the appearing objects

$$Z_k := \min\{c, R_k\} - g/n \tag{4.1}$$

are independent and identically distributed (i.i.d.) random variables. Note that R_k is $\mathcal{F}_{t_k}$-measurable such that H_T is $\mathcal{F}_{t_n}$-measurable. Since $t_n \leq T$, it holds $\mathcal{F}_{t_n} \subseteq \mathcal{F}_T$ such that H_T constitutes an $\mathcal{F}_T$-measurable claim. As before, let us denote the constant interest rate by $r > 0$. Then the price at time $t \leq T$ of a cliquet option with payoff H_T at maturity T is given by the discounted risk-neutral conditional expectation of the payoff, i.e.

$$C_t = e^{-r(T-t)} \mathbb{E}_{\mathbb{Q}}(H_T \mid \mathcal{F}_t).$$

Combining the latter equations, we obtain

$$C_0 = Ke^{-rT}\left(1+g+\mathbb{E}_{\mathbb{Q}}\left[\max\left\{0, \sum_{k=1}^{n} Z_k\right\}\right]\right) \tag{4.2}$$

which shows that the considered cliquet option with payoff H_T essentially is a plain-vanilla call option with strike zero written on the basket-style underlying $\sum_{k=1}^{n} Z_k$.

Proposition 4.1 (Cliquet option price)**.** *Let $k \in \{1, \ldots, n\}$ and consider the independent and identically distributed random variables $Z_k = \min\{c, R_k\} - g/n$ where $c \geq 0$ is the local cap, R_k is the return process given in (3.4) and g is the guaranteed rate at maturity. Denote the maturity time by T, the notional by K and the risk-less interest rate by r. Then the price at time zero of a cliquet option with payoff H_T can be represented as*

$$C_0 = Ke^{-rT}\left(1+g+\frac{n}{2}\mathbb{E}_{\mathbb{Q}}[Z_1] + \frac{1}{\pi}\int_{0+}^{\infty} \frac{1 - Re(\phi_Z(x))}{x^2} dx\right) \tag{4.3}$$

where Re denotes the real part and the characteristic function $\phi_Z(x)$ is defined via

$$\phi_Z(x) := \prod_{k=1}^{n} \phi_{Z_k}(x) = \prod_{k=1}^{n} \mathbb{E}_{\mathbb{Q}}\left[e^{ixZ_k}\right] = \left(\phi_{Z_1}(x)\right)^n = \left(\mathbb{E}_{\mathbb{Q}}\left[e^{ixZ_1}\right]\right)^n. \tag{4.4}$$

Proof. See the proof of Prop. 3.1 in [3], respectively of Prop. 3.1 in [8]. □

In the subsequent sections, we derive explicit expressions for $\phi_Z(x)$ and $\mathbb{E}_{\mathbb{Q}}[Z_1]$ appearing in the pricing formula (4.3). As before, we stick to the presumption of equidistant resetting times and set $\tau = t_k - t_{k-1}$ for all $k \in \{1, \ldots, n\}$ in the following.

4.1 Cliquet option pricing with distribution functions

Let us first apply a method involving probability distribution functions (cf. [3] and Section 3.1 in [8]). We initially investigate the treatment of $\phi_Z(x)$ defined in (4.4).

Proposition 4.2. *Let $Y_\tau \sim \mathcal{M}(\alpha, \beta, \delta\tau, (\mu + b)\tau)$ and suppose that $Z_k = \min\{c, R_k\} - g/n$ where $k \in \{1, \dots, n\}$. Then the characteristic function of Z_k under $\mathbb{Q}$ can be represented as*

$$\phi_{Z_k}(x) = e^{-ix(1+g/n)}\left(e^{ix(1+c)}\int_{1+c}^{\infty} f_{Y_\tau}(u)du + \int_{-\infty}^{0} f_{Y_\tau}(u)du + \int_{0}^{1+c} e^{ixu} f_{Y_\tau}(u)du\right) \tag{4.5}$$

where

$$f_{Y_\tau}(u) = \frac{(2\cos(\beta/2))^{2\delta\tau}}{2\pi\alpha\Gamma(2\delta\tau)} e^{\beta(u-(\mu+b)\tau)/\alpha}\left|\Gamma\left(\delta\tau + i\frac{u-(\mu+b)\tau}{\alpha}\right)\right|^2 \tag{4.6}$$

constitutes the probability density function of the Meixner–Lévy process Y given in (3.5) and b is the real-valued constant claimed in (3.3).

Proof. By similar arguments as in the proof of Prop. 3.2 in [8], we obtain

$$\phi_{Z_k}(x) = e^{-ix(1+g/n)}\left(e^{ix(1+c)} - ix\int_0^{1+c} e^{ixw}\mathbb{Q}(R_k \le w - 1)dw\right). \tag{4.7}$$

Using (3.6) and the definition of the distribution function, we get for the last integral in (4.7)

$$\int_0^{1+c} e^{ixw}\mathbb{Q}(R_k \le w - 1)dw = \int_0^{1+c}\int_{-\infty}^{\ln(w)} e^{ixw} f_{Y_\tau}(u)dudw$$

where $Y_\tau \sim \mathcal{M}(\alpha, \beta, \delta\tau, (\mu + b)\tau)$ is the Meixner–Lévy process given in (3.5) and $f_{Y_\tau}(u)$ constitutes the probability density function of Y_τ under $\mathbb{Q}$ claimed in (4.6). Applying Fubini's theorem and hereafter splitting up the resulting outer integral, we deduce

$$\int_0^{1+c} e^{ixw}\mathbb{Q}(R_k \le w - 1)dw = \int_{-\infty}^{0}\int_0^{1+c} e^{ixw} f_{Y_\tau}(u)dwdu + \int_0^{1+c}\int_u^{1+c} e^{ixw} f_{Y_\tau}(u)dwdu.$$

We next compute the emerging dw-integrals and finally substitute the resulting expression into (4.7) which yields (4.5). □

If we insert (4.5) into (4.4), we receive a representation for the characteristic function $\phi_Z(x)$. Let us proceed with the computation of $\mathbb{E}_{\mathbb{Q}}[Z_k]$.

Proposition 4.3. *Suppose that* $Z_k = \min\{c, R_k\} - g/n$ *where* $k \in \{1, \ldots, n\}$. *Then the first moment of* Z_k *under* $\mathbb{Q}$ *is given by*

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = -1 - \frac{g}{n} + (1+c)\int_{1+c}^{\infty} f_{Y_\tau}(u)du + \int_0^{1+c} u f_{Y_\tau}(u)du \tag{4.8}$$

where $f_{Y_\tau}(u)$ *is the probability density function of* Y_τ *under* $\mathbb{Q}$ *given in (4.6).*

Proof. In accordance to Prop. 2.4 in [4], we have

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = \frac{1}{i}\frac{\partial}{\partial x}\big(\phi_{Z_k}(x)\big)\big|_{x=0}. \tag{4.9}$$

A substitution of (4.5) into (4.9) instantly yields (4.8). □

As mentioned in Section 2, we recall that the cumulative distribution function (cdf) of the Meixner-Lévy process Y_t, i.e.

$$F_{Y_t}(x) := \int_{-\infty}^{x} f_{Y_t}(u)du$$

does not possess a closed form representation, but it can be computed efficiently with numerical methods. Also note that all integrals appearing in (4.5) and (4.8) are finite, since $f_{Y_\tau}(\cdot)$ constitutes a probability density function while the Meixner–Lévy process Y_τ possesses moments of all orders (cf. [13]).

4.2 *Cliquet option pricing with Fourier transform techniques*

There is an alternative method to derive expressions for $\mathbb{E}_{\mathbb{Q}}[Z_k]$, $\phi_Z(x)$ and C_0 involving Fourier transforms and the Lévy–Khinchin formula. In the following, we present this method which has firstly been proposed in [8] in a cliquet option pricing context.

Proposition 4.4. *Let* $Y_t \sim \mathcal{M}(\alpha, \beta, \delta t, (\mu+b)t)$ *be the Meixner–Lévy process considered in (3.5). Suppose that* $Z_k = \min\{c, R_k\} - g/n$ *where* $k \in \{1, \ldots, n\}$ *and let* $\vartheta > 0$ *be a finite real-valued dampening parameter. Then the first moment of* Z_k *under* $\mathbb{Q}$ *can be represented as*

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = c - \frac{g}{n} - \frac{1}{2\pi}\int_{\mathbb{R}} \frac{(c+1)^{1+\vartheta+iy}}{(\vartheta+iy)(1+\vartheta+iy)}\phi_{Y_\tau}(i\vartheta - y)dy \tag{4.10}$$

where the characteristic function ϕ_{Y_τ} *is given by*

$$\phi_{Y_\tau}(i\vartheta - y) = e^{-(\vartheta+iy)(\mu+b)\tau}\left(\frac{\cos(\beta/2)}{\cosh((i(\alpha\vartheta-\beta)-\alpha y)/2)}\right)^{2\delta\tau}. \tag{4.11}$$

Proof. The proof follows the same lines as the proof of Prop. 3.4 in [8]. From (4.1) and the equality

$$\min\{c, R_k\} = c - [c - R_k]^+$$

we deduce

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = c - g/n - \mathbb{E}_{\mathbb{Q}}\big[(c - R_k)^+\big].$$

Taking (3.4) into account, we next receive

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = c - g/n - \mathbb{E}_{\mathbb{Q}}\big[\big(c+1-e^{Y_\tau}\big)^+\big]$$

where $\tau = t_k - t_{k-1}$ and Y is the real-valued Meixner–Lévy process given in (3.5). With a finite and real-valued dampening parameter $\vartheta > 0$ we define the function

$$\varphi(u) := e^{\vartheta u}\big(c+1-e^u\big)^+.$$

Since $\varphi \in \mathcal{L}^1(\mathbb{R})$, its Fourier transform exists and reads as

$$\hat{\varphi}(y) = \frac{(c+1)^{1+\vartheta+iy}}{(\vartheta+iy)(1+\vartheta+iy)}.$$

Using the inverse Fourier transform along with Fubini's theorem, we get

$$\mathbb{E}_{\mathbb{Q}}\big[\big(c+1-e^{Y_\tau}\big)^+\big] = \mathbb{E}_{\mathbb{Q}}\big[e^{-\vartheta Y_\tau}\varphi(Y_\tau)\big] = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{\varphi}(y)\mathbb{E}_{\mathbb{Q}}\big[e^{-(\vartheta+iy)Y_\tau}\big]dy$$

which implies (4.10). The expression for the characteristic function ϕ_{Y_τ} given in (4.11) can directly be obtained by virtue of (2.9). □

Our argumentation in the proof of Proposition 4.4 motivates the following considerations.

Proposition 4.5. *Let $Y_t \sim \mathcal{M}(\alpha, \beta, \delta t, (\mu+b)t)$ be the Meixner–Lévy process presented in (3.5). Suppose that $Z_k = \min\{c, R_k\} - g/n$ with $k \in \{1, \ldots, n\}$ and $c \geq 0$. Then the characteristic function of Z_k under $\mathbb{Q}$ reads as*

$$\phi_{Z_k}(x) = e^{-ixg/n}\left(e^{ixc} + \int_{-\infty}^{\ln(1+c)} \big[e^{ix(e^u-1)} - e^{ixc}\big] f_{Y_\tau}(u)du\right) \tag{4.12}$$

where the probability density function f_{Y_τ} of Y_τ under $\mathbb{Q}$ is such as given in (4.6).

Proof. Similar computations as in the proof of Prop. 3.5 in [8] yield (4.12). □

There is an alternative method involving (4.9) to derive an expression for $\mathbb{E}_{\mathbb{Q}}[Z_k]$ which is presented in the following.

Corollary 4.6. *In the setup of Proposition 4.5, we receive the representation*

$$\mathbb{E}_{\mathbb{Q}}[Z_k] = c - \frac{g}{n} + \int_{-\infty}^{\ln(1+c)} \big[e^u - 1 - c\big] f_{Y_\tau}(u)du. \tag{4.13}$$

Proof. The claimed representation immediately follows from Eq. (3.16) in [8]. □

Inspired by the Fourier transform techniques applied in the proof of Proposition 4.4, we now focus on the derivation of an alternative representation for the cliquet option price C_0 given in (4.2). The corresponding result reads as follows.

Theorem 4.7 (Fourier transform cliquet option price)**.** *Let $k \in \{1, \dots, n\}$ and consider the independent and identically distributed random variables $Z_k = \min\{c, R_k\} - g/n$ where $c \geq 0$ is the local cap, g is the guaranteed rate at maturity and R_k is the return process defined in (3.4). For $n \in \mathbb{N}$ we set $\varrho := nc - g$ and denote the maturity time by T, the notional by K and the riskless interest rate by r. Let $Y_t \sim \mathcal{M}(\alpha, \beta, \delta t, (\mu + b)t)$ be the Meixner–Lévy process given in (3.5). Then the price at time zero of a cliquet option paying*

$$H_T = K\left(1 + g + \max\left\{0, \sum_{k=1}^{n} Z_k\right\}\right)$$

at maturity can be represented as

$$C_0 = Ke^{-rT}\left[1 + g + \int_{0^+}^{\infty} \frac{1 + iy\varrho - e^{iy\varrho}}{2\pi y^2} \times \left(1 + \int_{-\infty}^{\ln(1+c)} \left[e^{iy(e^u - 1 - c)} - 1\right] f_{Y_\tau}(u)du\right)^n dy\right] \tag{4.14}$$

where $f_{Y_\tau}(u)$ constitutes the probability density function claimed in (4.6).

Proof. The proof of Theorem 3.7 in [8] here applies equally, if we replace f_{X_τ} therein by f_{Y_τ}. □

5 Conclusions

In this paper, we investigated the pricing of a monthly sum cap style cliquet option with underlying stock price modeled by a geometric pure-jump Meixner–Lévy process. In Section 2, we compiled various facts on the Meixner distribution and the related class of stochastic Meixner–Lévy processes. In Section 3, we introduced a stock price model driven by a Meixner–Lévy process and established a customized structure preserving measure change from the risk-neutral to the physical probability measure. Moreover, we obtained semi-analytic expressions for the cliquet option price by using the probability distribution function of the driving Meixner–Lévy process in Section 4.1 and by an application of Fourier transform techniques in Section 4.2. To read more on cliquet option pricing in a jump-diffusion Lévy model, the reader is referred to the accompanying article [8].

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