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Autoregressive approaches to import–export time series II: a concrete case study

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Abstract The present work constitutes the second part of a two-paper project that, in particular, deals with an in-depth study of effective techniques used in econometrics in order to make accurate forecasts in the concrete framework of one of the major economies of the most productive Italian area, namely the province of Verona. It is worth mentioning that this region is indubitably recognized as the core of the commercial engine of the whole Italian country. This is why our analysis has a concrete impact; it is based on real data, and this is also the reason why particular attention has been taken in treating the relevant economical data and in choosing the right methods to manage them to obtain good forecasts. In particular, we develop an approach mainly based on vector autoregression where lagged values of two or more variables are considered, Granger causality, and the stochastic trend approach useful to work with the cointegration phenomenon.

Keywords Econometrics time series, autoregressive models, Granger causality, cointegration, stochastic nonstationarity, trends and breaks

1 Introduction

In this second part of a two-paper project, we move from theory of autoregressive, possibly multivalued, time series to the study of a concrete framework. In particular, exploiting precious economic data that the Commerce Chamber of Verona Province

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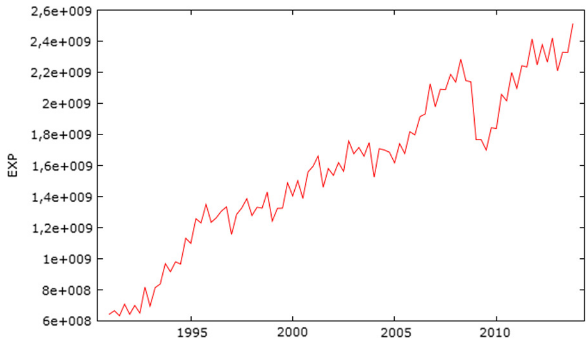

Fig. 1. Export of Verona

has put at our disposal, we successfully applied some of the relevant approaches introduced in [5] to find dependencies between economic factors characterizing the Province economy, then to make effective forecasts, very close to the real behavior of studied markets. The present part of the project is divided as follows: first, we consider an AR-approach to Verona import–export time series, then we provide a VAR model analysis of Verona relevant econometric data taken from various web databases such as Coeweb, Stockview, and Movimprese, and, within the last section, we compare such data with those coming from the whole Italian scenario. We would like to emphasize that all the theoretical background and related definitions can be retrieved from [5].

2 AR-approach to Verona import–export time series

In what follows, we shall apply techniques developed in previous sections to analyze our main empirical problem of forecasting export and import data for the Verona district, also using other variables such as active enterprises. These applications are based on Istat data retrieved from the database Coeweb.

2.1 EXP

We present a time series regression model in which the regressors are past values of the dependent variable, namely the Export data. We use 92 observations of variable EXP, quarterly data from 1991 to 2013 expressed in Euros. Figure 1 shows the related time series.

Looking at Fig. 1, we can see that the Verona export shows relatively smooth growth, although this decreases during the years 2008–2011. Decline in exports is likely caused by economic crisis broken out in Italy in those years. Although the curve may seem apparently growing, it is also possible to notice that there are periodic trends during the years under consideration. In fact, in the fourth quarter of 1992, the curve has a significant growth, then increases fairly linearly until about the second quarter of 1994, in which one can recognize a new increasing period that slightly more obvious than the previous one. This periodicity of 18 months can also be seen in other parts of the curve, but not after the beginning of the current economic crisis, where very likely there will be a structural break. In order to test the goodness of our qualitative analysis based on historical data, we used a software called GRETL, which is particularly useful to perform statical analysis of time series. The mean and standard deviation related to the quarter of this variable EXP are respectively $Mean = 1\,579\,900\,000$ € *and* $StandardDeviation = 499\,880\,000$ €, whereas the annual mean for EXP is $1\,579\,900\,000 \times 4 = 6\,319\,600\,000$ €. The first seven autocorrelations of EXP are $\rho_1 = corr(EXP_t, EXP_{t-1}) = 0.9718$, $\rho_2 = 0.9755$, $\rho_3 = 0.9450$, $\rho_4 = 0.9523$, $\rho_5 = 0.9165$, $\rho_6 = 0.9242$, $\rho_7 = 0.8931$. Previous entries show that inflation is strongly positively autocorrelated; in fact, the first autocorrelation is 0.97. The autocorrelation remains large even at a lag of six quarters. This means that an increase in export in one quarter tends to be associated with an increase in the next quarter. Autocorrelation starts to decrease from the lag of seventh quarters. In what follows, we report the output obtained testing for autoregressive models according to an increasing number of delays, from 1 to 6 delays, on the variable EXP, namely:

the AR(1) case: $EXP = 65\,090\,000 + 0.971606 EXP_{t-1}$

	Coefficient	Standard Error	t-Statistic	p-Value
const	6.50900e+007	2.35520e+007	2.7637	0.0069
EXP_{t-1}	0.971606	0.017392	55.8652	0.0000

SER	1.17e+08		
R^2	0.944426	Adjusted R^2	0.943802
AIC	3641.074	BIC	3646.096

the AR(2) case: $EXP = 57\,965\,600 + 0.409313 EXP_{t-1} + 0.573763 EXP_{t-2}$

	Coefficient	Standard Error	t-Statistic	p-Value
const	5.79656e+007	2.92851e+007	1.9794	0.0509
EXP_{t-1}	0.409313	0.0920617	4.4461	0.0000
EXP_{t-2}	0.573763	0.105188	5.4546	0.0000

SER	97 111 006		
R^2	0.60913	Adjusted R^2	0.960014
AIC	3568.804	BIC	3576.303

the AR(3) case: $EXP = 54\,025\,100 + 0.618705 EXP_{t-1} + 0.726958 EXP_{t-2} - 0.366510 EXP_{t-3}$

	Coefficient	Standard Error	t-Statistic	p-Value
const	5.40251e+007	2.26874e+007	2.3813	0.0195
EXP_{t-1}	0.618705	0.109790	5.6353	0.0000
EXP_{t-2}	0.726958	0.063352	11.4749	0.0000
EXP_{t-3}	−0.366510	0.115843	−3.1639	0.0022

SER	91 264 682		
R^2	0.964681	Adjusted R^2	0.963435
AIC	3519.089	BIC	3529.044

the AR(4) case: $EXP = 54\,498\,000 + 0.748057 EXP_{t-1} + 0.466614 EXP_{t-2} - 0.592869 EXP_{t-3}$

	Coefficient	Standard Error	t-Statistic	p-Value
const	5.44980e+007	2.43509e+007	2.2380	0.0279
EXP_{t-1}	0.748057	0.142495	5.2497	0.0000
EXP_{t-2}	0.466614	0.075211	6.2041	0.0000
EXP_{t-3}	−0.592869	0.156045	−3.7993	0.0003
EXP_{t-4}	0.361048	0.065852	5.4827	0.0000

SER	86 223 417		
R^2	0.967898	Adjusted R^2	0.966351
AIC	3470.537	BIC	3482.924

the AR(5) case: $EXP = 56\,242\,200 + 0.870848EXP_{t-1} + 0.247032EXP_{t-2} - 0.417031EXP_{t-3} + 0.648298EXP_{t-4} - 0.372917EXP_{t-5}$

	Coefficient	Standard Error	t-Statistic	p-Value
const	5.62422e+007	2.12088e+007	2.6518	0.0096
EXP_{t-1}	0.870848	0.135548	6.4246	0.0000
EXP_{t-2}	0.247032	0.096569	2.5581	0.0124
EXP_{t-3}	−0.417031	0.178982	−2.3300	0.0223
EXP_{t-4}	0.648298	0.105669	6.1352	0.0000
EXP_{t-5}	−0.372917	0.119834	−3.1119	0.0026

SER	80 872 743		
R^2	0.970976	Adjusted R^2	0.969185
AIC	3420.938	BIC	3435.733

and the AR(6) case:

$$EXP = 55434600 + 1.01304EXP_{t-1} + 0.00610464EXP_{t-2} - 0.251406EXP_{t-3} + 0.542831EXP_{t-4} - 0.737681EXP_{t-5} + 0.408104EXP_{t-6} \quad (1)$$

	Coefficient	Standard Error	t-Statistic	p-Value
const	5.54346e+007	2.23371e+007	2.4817	0.0152
EXP_{t-1}	1.01304	0.12541	8.0777	0.0000
EXP_{t-2}	0.006105	0.107043	0.0570	0.9547
EXP_{t-3}	−0.251406	0.131646	−1.9097	0.0598
EXP_{t-4}	0.542831	0.116130	4.6743	0.0000
EXP_{t-5}	−0.737681	0.104151	−7.0828	0.0000
EXP_{t-6}	0.408104	0.089469	4.5614	0.0000

SER	75 057 009		
R^2	0.974384	Adjusted R^2	0.972438
AIC	3369.763	BIC	3386.943

We estimate the AR order of our autoregression related to obtained numerical results using both BIC and AIC information criteria (see Table 1).

Table 1. BIC, AIC, Adjusted R^2, and SER for the six AR models

p	$BIC(p)$	$AIC(p)$	Adjusted $R^2(p)$	$SER(p)$
1	3646.096	3641.074	0.943802	117000000
2	3576.303	3568.804	0.960014	97111006
3	3529.044	3519,089	0.963435	91264682
4	3482.924	3470.537	0.966351	86223417
5	3435.733	3420.938	0.969185	80872743
6	3386.943	3369.763	0.972438	75057009

Table 2. Large-sample critical values of the augmented Dickey–Fuller statistic

Deterministic Regressors	10 %	5 %	1 %
Intercept only	–2.57	–2.86	–3.43
Intercept and time trend	–3.12	–3.41	–3.96

Both BIC and AIC are the smallest in the AR(6) model (from the seventh delay onwards the criteria begin to increase); we conclude that the best estimate of the lag length is 6, hence supporting our qualitative analysis. Previous data from Table 1 indicate that as the number of lags increases, the *Adjusted* R^2 increases, and the SER decreases. R^2, *Adjusted* R^2, *and SER* measure how well the OLS estimate of the multiple regression line describes the data. The standard error of the regression (SER) estimates the standard deviation of the error term, and thus, it is a measure of spread of the distribution of a variable Y around the regression line. The regression R^2 is the fraction of the sample variance of Y explained by (or predicted by) the regressors, the R^2 increases whenever a regressor is added, unless the estimated coefficient on the added regressor is exactly zero. An increase in the R^2 does not mean that adding a variable actually improves the fit of the model, so the R^2 gives an inflated estimate of how well the regression fits the data. One way to correct this is to deflate or reduce the R^2 by some factor, and this is what the *Adjusted* R^2 does, which is a modified version of R^2 that does not necessarily increase when a new regressor is added. As seen by numerical output in Table 1, the increase in *Adjusted* R^2 is large from one to two lags, smaller from two to three, and quite small from three to four and in the next lags. Exploiting the results obtained for the AIC/BIC analysis, we can determine how large the increase in the *Adjusted* R^2 must be to justify including the additional lag. In the AR(6) model of Eq. (1), the coefficients of EXP_{t-1}, EXP_{t-4}, EXP_{t-5}, and EXP_{t-6} are statically significant at the 1% significance level because their p-value is less than 0.01, and the t-statistic exceeds the critical value. The constant, however, is statically significant at the 5% significance. The coefficient of EXP_{t-3} is statically significant at the 10% significance, and the coefficient of EXP_{t-2} is not statically significant. In particular, the 95% confidence intervals for these coefficient are as follows:

Variable	Coefficient	95% Confidence Interval	
const	5.54346e+007	1.09738e+007	9.98955e+007
EXP_{t-1}	1.01304	0.76341	1.26266
EXP_{t-2}	0.006105	−0.206959	0.219168
EXP_{t-3}	−0.251406	−0.513441	0.010627
EXP_{t-4}	0.542831	0.311680	0.773981
EXP_{t-5}	−0.737681	−0.944989	−0.530374
EXP_{t-6}	0.408104	0.230022	0.586187

In order to check whether the EXP variable has a trend component or not, we test the null hypothesis that such a trend actually exists against the alternative EXP being stationary, by performing the ADF test for a unit autoregressive root. Large-sample critical values of the augmented Dickey–Fuller statistic yield the following ADF regression with six lags of EXP_t, where the subscript t indicates a particular quarter considered:

$$\begin{aligned}\widehat{\Delta EXP_t} &= 55\,434\,600 + \delta EXP_{t-1} + \gamma_1 \Delta EXP_{t-1} + \gamma_2 EXP_{t-2} \\ &\quad + \gamma_3 \Delta EXP_{t-3} + \gamma_4 \Delta EXP_{t-4} + \gamma_5 \Delta EXP_{t-5} + \gamma_6 \Delta EXP_{t-6}. \qquad (2)\end{aligned}$$

Table 3. Critical values of QLR statistic with 15% truncation

Number of restrictions	10 %	5 %	1 %
7	2.84	3.15	3.82

The ADF t-statistic is the t-statistic testing the hypothesis that the coefficient on EXP_{t-1} is zero; this is $t = -1.23$. From Table 2, the 5% critical value is -2.86. Because the ADF statistic of -1.23 is less negative than -2.86, the test does not reject the null hypothesis at the 5% significance level. Based on the regression in Eq. (2), we therefore cannot reject the null hypothesis that export has a unit autoregressive root, that is, that export contains a stochastic trend, against the alternative that it is stationary. If instead the alternative hypothesis is that Y_t is stationary around a deterministic linear trend, then the ADF t-statistic results in $t = -4.07$, which is less than -3.41 (from Table 2). Hence, we can reject the null hypothesis that export has a unit autoregressive root. We proceed with a test QLR, which provides a way to check whether the export curve has been stable in the period from 1993 to 2010. Specifically, we focus on whether there have been changes in the coefficients of the lagged values of export and of the intercept in the AR(6) model specification in Eq. (1) containing six lags of EXP_t. The Chow F-statistics (see, e.g., [7, Sect. 5.3.3]) tests the hypothesis that the intercept and the coefficients of $EXP_{t-1}, \ldots, EXP_{t-6}$ in Eq. (1) are constant against the alternative that they break at a given date for breaks in the central 70% of the sample. The F-statistic is computed for break dates in the central 70% of the sample because for the large-sample approximation to the distribution of the QLR statistic to be a good one, the subsample endpoints cannot be too close to the beginning or to the end of the sample, so we decide to use 15% trimming, that is, to set $\tau_0 = 0.15T$ and $\tau_1 = 0.85T$ (rounded to the nearest integer). Each F-statistic tests seven restrictions. Restrictions on the coefficients equaled to zero under the null hypothesis (see [5, Sect. 2.4]), and since in our case we have the coefficients of the six delays and the intercept, we get seven restrictions. The largest of these F-statistics is 13.96, which occurs in 2010:I (the first quarter of 2010); this is the QLR statistic. The critical value for seven restrictions is presented in Table 3.

The previously reported values indicate that the hypothesis of stable coefficients is rejected at the 1% significance level. Thus, there is an evidence that at least one of these seven coefficients changed over the sample. These results also confirm the assumptions that we made earlier since the year 2010 coincides with an increasing import of the financial crisis before arriving at a partial economic recovery. A forecast of Verona export in 2014:I using data through 2013:IV can be then based on our established AR(6) model of export, which gives

$$EXP = 55\,434\,600 + 1.01304 EXP_{t-1} + 0.00610464 EXP_{t-2} - 0.251406 EXP_{t-3} + 0.542831 EXP_{t-4} - 0.737681 EXP_{t-5} + 0.408104 EXP_{t-6}.$$

Therefore, substituting the values of export into each of the four quarters of 2013, plus the two last quarters of 2012, we have

$$\widehat{EXP}_{2014:I|2013:IV} = 55\,434\,600 + 1.013 EXP_{2013:\mathrm{IV}} + 0.006 EXP_{2013:\mathrm{III}} - 0.251 EXP_{2013:\mathrm{II}} + 0.543 EXP_{2013:\mathrm{I}}$$

$$
\begin{aligned}
&\quad - 0.738 EXP_{2012:IV} + 0.408 EXP_{2012:III} \\
&= 55\,434\,600 + 1.013 \times 2\,511\,098\,163 + 0.006 \times 2\,326\,958\,115 \\
&\quad - 0.251 \times 2\,329\,551\,351 + 0.543 \times 2\,209\,212\,521 \\
&\quad - 0.738 \times 2\,420\,606\,501 + 0.408 \times 2\,265\,903\,940 \\
&\cong 2\,366\,137\,617 \text{ €},
\end{aligned}
$$

so that, for 2014:II, we obtain

$$
\begin{aligned}
\widehat{EXP}_{2014:II|2014:I} &= 55\,434\,600 + 1.013 EXP_{2014:I} + 0.006 EXP_{2013:IV} \\
&\quad - 0.251 EXP_{2013:III} + 0.543 EXP_{2013:II} \\
&\quad - 0.738 EXP_{2012:I} + 0.408 EXP_{2012:IV} \\
&= 55\,434\,600 + 1.013 \times 2\,366\,137\,617 + 0.006 \times 2\,511\,098\,163 \\
&\quad - 0.251 \times 2\,326\,958\,115 + 0.543 \times 2\,329\,551\,351 \\
&\quad - 0.738 \times 2\,209\,212\,521 + 0.408 \times 2\,420\,606\,501 \\
&\cong 2\,505\,454\,123 \text{ €},
\end{aligned}
$$

and forecasts for all 2014 quarters are as follows:

Quarter	*Forecast*	*Error*
2014:I	2 366 130 000	75 057 000
2014:II	2 505 450 000	106 841 000
2014:III	2 422 950 000	131 981 000
2014:IV	2 527 660 000	145 016 000

It is worth mentioning that the forecast error increases as the number of considered quarters increases. Figure 2 shows, through a graph, forecasts since 2002 in sample and forecasts for 2014, highlighting the confidence intervals.

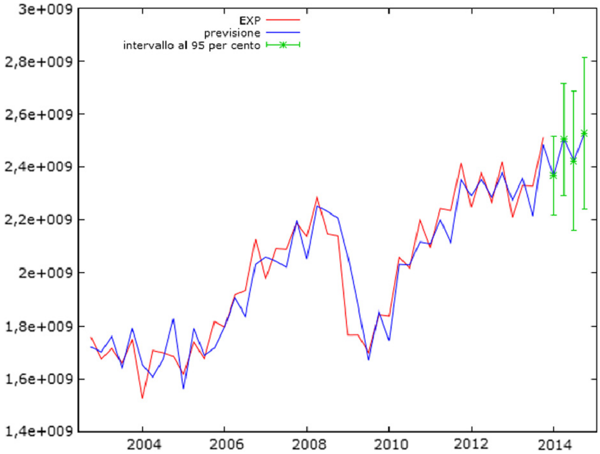

Fig. 2. Forecasts of Verona export

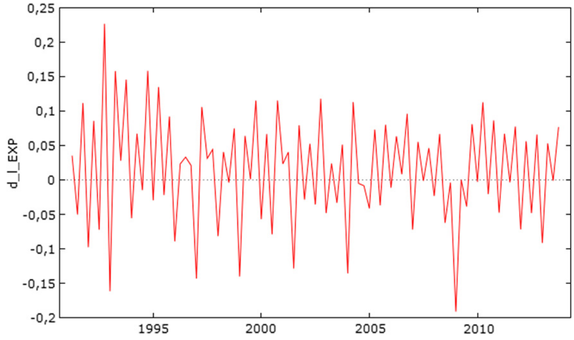

Fig. 3. Rate of growth in exports

2.2 ΔEXP

It is also useful to analyze the time series of the growth rate in exports that we denoted by ΔEXP. Economic time series are often analyzed after computing their logarithms or the changes in their logarithms. One reason for this is that many economic series exhibit growth that is approximately exponential, that is, over the long run, the series tends to grow by a certain percentage per year on average, and hence the logarithm of the series grows approximately linearly. Another reason is that the standard deviation of many economic time series is approximately proportional to its level, that is, the standard deviation is well expressed as a percentage of the level of the series; hence, if this is the case, the standard deviation of the logarithm of the series is approximately constant. It follows that it turns to be convenient to work with the variable $\Delta EXP_t = \ln(EXP_t) - \ln(EXP_{t-1})$. Taking into account the data shared in Fig. 3, we retrieve the following information:

$$\textit{Mean on a quarterly basis} = 0.014958 = 1.49\%$$

$$\textit{Standard Deviation on a quarterly basis} = 0.079272 = 7.93\%$$

$$\textit{Average Growth Rate on a yearly basis} = 0.014958 \times 4 = 0.059832 = 5.98\%$$

The first four autocorrelations of ΔEXP are $\rho_1 = -0.6133$, $\rho_2 = 0.5698$, $\rho_3 = -0.6100$, $\rho_4 = 0.7029$.

Even if it might seem contradictory that the level of export is strongly positively correlated but its change is negatively correlated, we have to consider that such values measure different things. The strong positive autocorrelation in export reflects the long-term trends in export; in contrast, the negative autocorrelation of the change of export means that, on average, an increase in export in one quarter is associated with a decrease in export in the next one. Analogously to what we have seen in Section 2.1, we perform an AIC/BIC analysis for ΔEXP obtaining that the best choice for the lag lay is 4, so that we have

$$\begin{aligned}\Delta EXP = 0.0128189 - 0.173627\Delta EXP_{t-1} + 0.0996175\Delta EXP_{t-2}\\ - 0.189882\Delta EXP_{t-3} + 0.416414\Delta EXP_{t-4}.\end{aligned}$$

	Coefficient	Standard Error	t-Statistic	p-Value
const	0.0128189	0.0077887	1.6458	0.1036
ΔEXP_{t-1}	−0.173627	0.119987	−1.4470	0.1517
ΔEXP_{t-2}	0.099618	0.100542	0.9908	0.3247
ΔEXP_{t-3}	−0.189882	0.096363	−1.9705	0.0522
ΔEXP_{t-4}	0.416414	0.094464	4.4081	0.0000

SER	0.052736		
R^2	0.576787	Adjusted R^2	0.556142
AIC	–260.2402	BIC	–247.9107

In our AR(4) model, the coefficients of ΔEXP_{t-4} are statically significant at the 1% significance level because their p-value is less than 0.01 and the t-statistic exceeds the critical value. The coefficient of ΔEXP_{t-3} is statically significant at the 10% significance. The constant and the other coefficients are not statically significant. Even when the information criteria are very low, this is not a good model because R^2 and *Adjusted* R^2 are relatively small. So this AR(4) model turns out to be not very useful to predict the growth rate in exports. Figure 3 shows that the frequency in this case is annual; moreover, an increase in ΔEXP in one quarter is associated with a decrease in the next one. In this case, the results of ADF test allow us to reject the null hypothesis that rate of growth in export has a unit autoregressive root both with the alternative hypothesis of stationarity and of stationarity around a deterministic linear trend. It follows that the QLR statistic is 5.02, which occurs in 2009:I, and hence the hypothesis that the coefficients are stable is rejected at the 1% significance level. Again, the results of the software GRETL confirm that the crisis of recent years has greatly affected the exports from Verona. Consequently, by the results obtained we have that the forecast of ΔEXP for 2014, given in the table

Quarter	*Forecast*	*Error*
2014:I	–4.86%	0.052736
2014:II	5.11%	0.053525
2014:III	–1.58%	0.053961
2014:IV	6.16%	0.055304

and also sketched in Fig. 4, is not very accurate, and the predictions do not perceive the lower peaks of the variable, which is confirmed by the low value of R^2.

2.3 *IMP*

We now turn to the empirical problem to predict Verona import by analyzing its historical series. We present an autoregressive model that uses the history of Verona import to forecast its future. We use 92 observations of variable import, quarterly data from 1991 to 2013 expressed in Euros. Figure 5 shows the time series.

Looking at Fig. 5, we can see that Verona import shows relatively smooth growth, although this decreases during the years 2008–2011; the curve is very similar to the time series of export, and hence it is reasonable to deduce that decline in import is likely caused by economic crisis broken out in Italy in those years. Although the curve may seem apparently growing, periodic trends appear during years under consideration. This curve has an annual periodicity. Looking at a minimum of the curve, exactly one year later, another minimum exists. The mean and standard deviation on a quarterly basis for IMP are $Mean = 2\,177\,300\,000$ € and $StandardDeviation = 697\,420\,000$ €, whereas the annual mean export is $2\,177\,300\,000 \times 4 =$

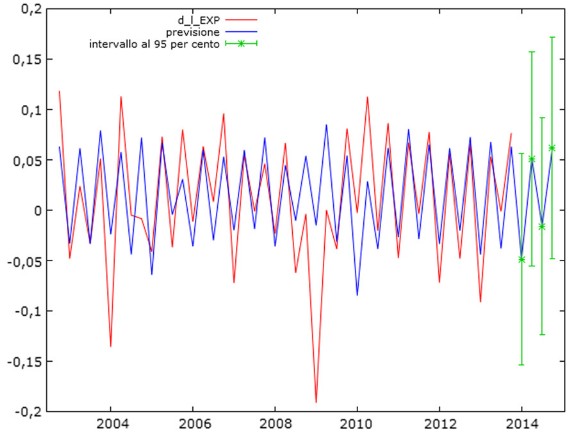

Fig. 4. Forecasts of ΔEXP

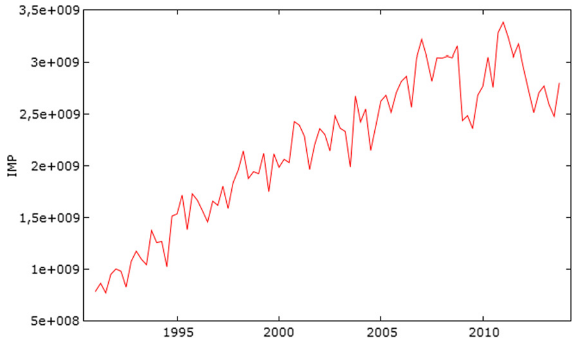

Fig. 5. Import of Verona

8 709 200 000 €. The first five IMP autocorrelation values are $\rho_1 = 0.9424$, $\rho_2 = 0.9280$, $\rho_3 = 0.9060$, $\rho_4 = 0.9260$, $\rho_5 = 0.8750$. These entries show that inflation is strongly positively autocorrelated; in fact, the first autocorrelation is 0.94. The autocorrelation remains large even at the lag of four quarters. This means that an increase in import in one quarter tends to be associated with an increase in the next quarter. Autocorrelation, as expected, starts to decrease starting from the lag of five quarters. As with the variable EXP, we estimated the AR order of an autoregression in IMP using both the AIC and BIC information criteria, finally obtaining that the optimal lag length is 4.

	Coefficient	Standard Error	t-Statistic	p-Value
const	1.90005e+008	6.15140e+007	3.0888	0.0027
IMP_{t-1}	0.499665	0.0997006	5.0117	0.0000
IMP_{t-2}	0.155637	0.0746261	2.0856	0.0401
IMP_{t-3}	−0.154911	0.0881396	−1.7576	0.0825
IMP_{t-4}	0.434062	0.0827892	5.2430	0.0000

SER	198 000 000		
R^2	0.911613	Adjusted R^2	0.907354
AIC	3616.812	BIC	3629.198

Therefore, we have

$$IMP = 190\,005\,000 + 0.499665 IMP_{t-1} + 0.155637 IMP_{t-2} - 0.154911 IMP_{t-3} + 0.434062 IMP_{t-4}. \tag{3}$$

We check now if the model has a trend. The null hypothesis that Verona import has a stochastic trend can be tested against the alternative that it is stationary by performing the ADF test for a unit autoregressive root. The ADF regression with four delays of IMP gives

$$\widehat{\Delta IMP}_t = 190005000 + \delta IMP_{t-1} + \gamma_1 \Delta IMP_{t-1} + \gamma_2 IMP_{t-2} + \gamma_3 \Delta IMP_{t-3} + \gamma_4 \Delta IMP_{t-4}. \tag{4}$$

The ADF t-statistic is the t-statistic testing the hypothesis that the coefficient on IMP_{t-1} is zero, and it turns to be $t = -1.78$. From Table 2, the 5% critical value is -2.86. Because the ADF statistic of -1.78 is less negative than -2.86, the test does not reject the null hypothesis at the 5% significance level. We therefore cannot reject the null hypothesis that import has a unit autoregressive root, that is, that import contains a stochastic trend, against the alternative that it is stationary. If the alternative hypothesis is that Y_t is stationary around a deterministic linear trend, then the ADF t-statistic results in $t = -2.6$, which is less negative than -3.41. So, in this case, we also cannot reject the null hypothesis that export has a unit autoregressive root.

We proceed with a QLR test, which provides a way to check whether the import curve has been stable during the years sparing from 1993 to 2010. The Chow F-statistic tests the hypothesis that the intercept and the coefficients at $IMP_{t-1}, \ldots,$ IMP_{t-4} in Eq. (3) are constant against the alternative that they break at a given date for breaks in the central 70% of the sample. Each F-statistic tests five restrictions. The largest of these F-statistics is 10.26, which occurs in 1995:III; the critical values for the five-restriction model at different levels of significance are given in Table 4. These values indicate that the hypothesis that the coefficients are stable is rejected at the 1% significance level. Thus, there is an evidence that at least one of these five coefficients changed over the sample; namely, we have a structural break, which might be caused by the devaluation that the Lira currency experienced during the period 1992–1995. According to the previous analysis, the predictions of import of Verona for the year 2014 are as follows:

Quarter	*Forecast*	*Error*
2014:I	2 775 360 000	197 957 000
2014:II	2 752 530 000	197 957 000
2014:III	2 639 510 000	235 388 000
2014:IV	2 721 670 000	236 693 000

Table 4. Critical values of QLR statistic with 15% truncation

Number of restrictions	10 %	5 %	1 %
5	3.26	3.66	4.53

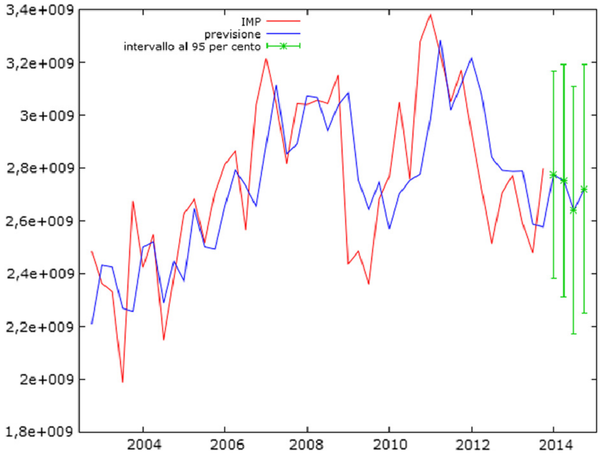

Fig. 6. Forecasts of Verona import

Table 5. Autocorrelations of ΔIMP

j	1	2	3	4	5	6
ρ_j	–0.4240	0.0631	–0.3910	0.6721	–0.3844	0.0743

They result in a slight increase for the next year, as shown by Fig. 6.

2.4 ΔIMP

The fourth variable of interest is represented by the logarithm of the ratio between consecutive values of IMP, that is,

$$\Delta IMP_t = \ln(IMP_t) - \ln(IMP_{t-1}) = \ln\left(\frac{IMP_t}{IMP_{t-1}}\right).$$

The first six autocorrelations values of ΔIMP are presented in Table 5.

In the case of the growth rate of export, the negative autocorrelation of the change of import means that, on average, an increase in import in one quarter is associated with a decrease in the next one. From the fifth lag, autocorrelation starts to be less significant. So, it can be easily seen from Fig. 7 and the autocorrelations in Table 5 that the right estimate of the lag length is 4. The consequent AR(4) model reads as follows:

$$\begin{aligned}\Delta IMP = {} & 0.0128189 - 0.173627\Delta IMP_{t-1} + 0.0996175\Delta IMP_{t-2} \\ & - 0.189882\Delta IMP_{t-3} + 0.416414\Delta IMP_{t-4}\,, \end{aligned} \tag{5}$$

and the following Fig. 7 shows the time series of ΔIMP, and we can see how an increase in import in one quarter is associated with a decrease in the next one.

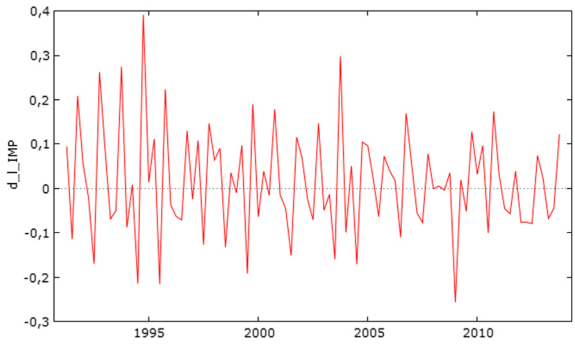

Fig. 7. Rate of growth in imports

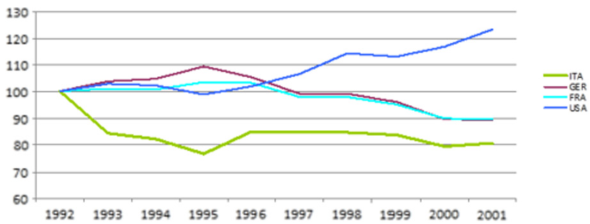

Fig. 8. Evolution of the Real Exchange Rate (index numbers: 1992 = 100) (*Source: FMI*)

	Coefficient	Standard Error	t-Statistic	p-Value
const	0.0161472	0.0110277	1.4642	0.1470
ΔIMP_{t-1}	−0.326437	0.0950214	−3.4354	0.0009
ΔIMP_{t-2}	−0.224760	0.0878146	−2.5595	0.0123
ΔIMP_{t-3}	−0.280232	0.0960526	−2.9175	0.0046
ΔIMP_{t-4}	0.431620	0.0894247	4.8266	0.0000

SER	0.083791		
R^2	0.531621	Adjusted R^2	0.508773
AIC	–179.6755	BIC	–167.3459

The QLR statistic for AR(4) model in Eq. (5) is 22.58, which occurs in 1995:II. This value indicates that the hypothesis that the coefficients are stable is rejected at the 1% significance level. As for imports, we can associate this structural break to the last crisis of Lira occurred in that period. We observe the dynamics of the real effective exchange rate in Fig. 8.

As shown in Fig. 9, the devaluation of the Lira has produced some benefits for the growth of Italian exports (goods and services), especially looking at analogous economical data for Germany and France.

As shown in Fig. 10, the devaluation of the Lira did not stop the value of imports, but you can still easily perceive the rupture of 1995.

2.5 Active Enterprises

We would like also to briefly analyze the variable “Active Enterprises” ($ACTE_t$), namely the time series with quarterly data from 1995 to 2013, where each obser-

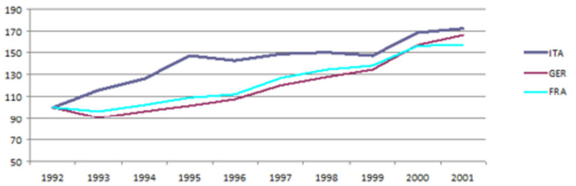

Fig. 9. Growth of Exports of Goods and Services (index numbers: 1992 = 100; correct values with the GDP deflator) (*Source: World Bank data*)

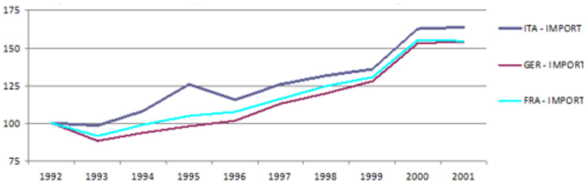

Fig. 10. Growth of Imports of Goods and Services (index numbers: 1992 = 100; correct values with the GDP deflator) (*Source: World Bank data*)

vation is the number of firms operating in a given quarter in the province of Verona. With the software GRETL we obtain the AR(4) model

$$ACTE = 9535.97 + 1.02210 ACTE_{t-1} - 0.173385 ACTE_{t-2} + 0.0152586 ACTE_{t-3} + 0.0280194 ACTE_{t-4} \,. \tag{6}$$

The *Adjusted* R^2 of this regression is 0.94, and the QLR statistic is 37.52, which occurs in 2011:I. This value indicates that the hypothesis that the coefficients are stable is rejected at the 1% significance level. Also, for the variable $ACTE_t$, we can conclude that the number of active businesses were affected by the crisis of those years. However, the ADF t-statistic for this variable does not reject the null hypothesis, so we cannot reject the fact that the time series of the numbers of active enterprises has a unit autoregressive root, that is, that $ACTE_t$ contains a stochastic trend, against the alternative that it is stationary. From Fig. 11 we can see that the curve has a quite regular annual pattern and that active enterprises tend to decline in the first quarter of each year and then return generally to grow. It is worth to mention the drastic rise of the curve during the first period of the time interval under consideration. Such an increase has been caused by a particular type of bureaucratic constraints, namely by a sort of forced registration imposed to a rather large set of farms companies previously not obliged to be part of the companies register. Such a norm has been introduced in two steps, first by a simple communication (1993), and later in the form of legal disclosure (2001).

3 VAR models analysis of Verona data

In this section, we apply the theory developed in the fourth chapter to analyze the set of Verona import and export time series. Therefore, we consider a VAR model for

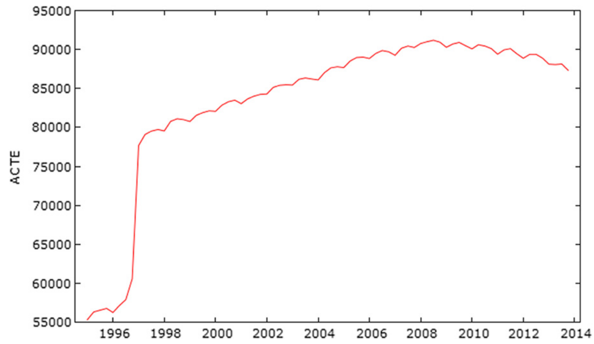

Fig. 11. $ACTE_t$

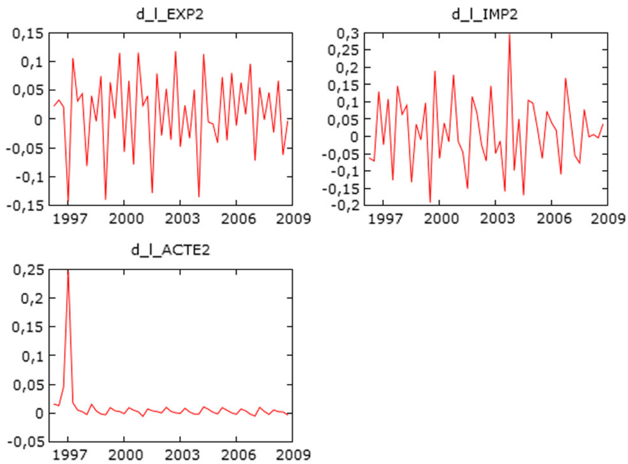

Fig. 12. Multiple graph for ΔEXP_t, ΔEXP_t and $\Delta ACTE_t$

exports (EXP_t), imports (IMP_t), and active companies ($ACTE_t$) in Verona, and each of such variables is characterized by time series constituted by quarterly data from 1995 to 2013.

3.1 First model: stationary variables

As we saw in Chapter 2, the import end export of Verona are subject to a stochastic trend, so that it is appropriate to transform it by computing its logarithmic first differences in order to obtain stationary variables. Figure 12 shows a multiple graph for the time series of ΔEXP_t, ΔIMP_t, and $\Delta ACTE_t$.

The VAR for ΔEXP_t, ΔIMP_t, and $\Delta ACTE_t$ consists of three equations, each of which is characterized by a dependent variable, namely by ΔEXP_t, ΔIMP_t, and $\Delta ACTE_t$, respectively. Because of the apparent breaks in considered time series for the years 1995 and 2010, the VAR is estimated using data from 1996:I to 2008:IV. The number of lags of this model are obtained through information criteria BIC and AIC using the software GRETL, which gives the results in Table 6, where the asterisks indicate the best (or minimized) of the respective information criteria.

Table 6. VAR lag lengths

p	$AIC(p)$	$BIC(p)$
1	–13.610968	–13.119471
2	–14.685333	–13.825212*
3	–14.572491	–13.343746
4	–14.747567	–13.150199
5	–14.974180	–13.008189
6	–15.160238*	–12.825624
7	–15.048342	–12.345105
8	–15.047682	–11.975822

The smallest AIC has been obtained considering six lags; indeed, the BIC estimation of the lag length is $\hat{p} = 2$. We decide to choose two delays because, for $\hat{p} = 6$, we have a VAR with three variables and six lags, so we will have 19 coefficients (eight lags with three variables each, plus the intercept) in each of the three equations, with a total of 57 coefficients, and we saw in [5, Sect. 4.2] that estimation of all these coefficients increases the amount of the forecast estimation error, resulting in a deterioration of the accuracy of the forecast itself. We also prefer consider the BIC estimation for its consistency; however, the AIC overestimate p (see [5, Sect. 2.2]. Estimating the VAR model with GRETL produces the following results:

$$\begin{aligned}\Delta EXP_t &= 0.0014 - 0.44\Delta EXP_{t-1} - 0.14\Delta EXP_{t-2} - 0.19\Delta IMP_{t-1}\\ &\quad + 0.21\Delta IMP_{t-2} - 0.15\Delta ACTE_{t-1} + 0.35\Delta ACTE_{t-2},\\ \Delta IMP_t &= 0.0222 - 0.5\Delta EXP_{t-1} + 0.57\Delta EXP_{t-2} - 0.38\Delta IMP_{t-1}\\ &\quad - 0.46\Delta IMP_{t-2} + 0.09\Delta ACTE_{t-1} + 0.2\Delta ACTE_{t-2},\\ \Delta ACTE_t &= 0.0043 + 0.02\Delta EXP_{t-1} + 0.12\Delta EXP_{t-2} + 0.07\Delta IMP_{t-1}\\ &\quad - 0.02\Delta IMP_{t-2} + 0.23\Delta ACTE_{t-1} + 0.02\Delta ACTE_{t-2}.\end{aligned} \tag{7}$$

In the first equation (ΔEXP_t) of VAR system (7), we have the coefficients of ΔEXP_{t-1}, ΔIMP_{t-2}, and $\Delta ACTE_{t-2}$, which are statically significant at the 1% significance level because their p-value is less than 0.01 and the t-statistic exceeds the critical value. The constant and the coefficients of ΔIMP_{t-1}, however, are statically significant at the 5% significance, and the other coefficients are not statically significant. The *Adjusted* R^2 is 0.53. In the second equation (ΔIMP_t) of VAR system (7), we have the coefficients of ΔEXP_{t-1}, ΔEXP_{t-2}, ΔIMP_{t-1}, and ΔIMP_{t-2}, which are statically significant at the 1% significance level. The constant, however, is statically significant at the 10% significance, and the other coefficients are not statically significant. The *Adjusted* R^2 is 0.45. In the last equation of (7), we have only the constant statically significant, at the 5% level. The *Adjusted* R^2 is −0.04. These VAR equations can be used to perform Granger causality tests. The results of this test for the first equation of (7) are as follows:

Variable	Test F	p-Value
ΔIMP_t	12.464	0.0001
$\Delta ACTE_t$	8.2240	0.0010

The F-statistic testing the null hypothesis that the coefficients of ΔIMP_{t-1} and

ΔIMP_{t-2} are zero in the first equation is 12.46 with p-value 0.0001, which is less than 0.01. Thus, the null hypothesis is rejected at the level of 1%, so we can conclude that the growth rate in Verona import is a useful predictor for the growth rate in export, namely ΔIMP_t Granger-causes ΔEXP_t. Also, $\Delta ACTE_t$ Granger-causes the change in export at the 1% significance level. The results for the second equation of (7) are as follows:

Variable	Test F	p-Value
ΔEXP_t	22.766	0.0000
$\Delta ACTE_t$	1.5894	0.2161

For the ΔIMP_t equation, we can also conclude that the growth rate in Verona export is a useful predictor for the growth rate in import, but the change in the number of active enterprises is not. The results for the last equation of (7) are as follows:

Variable	Test F	p-Value
ΔEXP_t	1.0897	0.3456
ΔEXP_t	1.6413	0.2059

The F-statistic testing the null hypothesis that the coefficients of ΔEXP_{t-1} and ΔEXP_{t-2} are zero in the first equation is 1.09 with p-value 0.34, which is greater than 0.10. Thus, the null hypothesis is not rejected, so we can conclude that the growth rate in Verona import is not a useful predictor for the growth rate in active enterprises, namely, ΔIMP_t does not Granger-cause $\Delta ACTE_t$. The F-statistic testing the hypothesis that the coefficients of the two lags of ΔEXP_t are zero is 1.64 with p-value of 0.2; thus, ΔEXP_t also does not Granger-cause $\Delta ACTE_t$ at the 10% significance level. Forecasts of the three variables in system (7) are obtained exactly as discussed in the univariate time series models, but in this case, the forecast of ΔEXP_t, we also consider past values of ΔIMP_t and $\Delta ACTE_t$.

Table 7. Forecasts of ΔEXP_t

Quarter	ΔEXP_t	Forecast	Error	95% Confidence Interval	
2009:1	–0.02329	0.018816	0.044544	−0.071077	0.108709
2009:2	0.06674	0.017759	0.052249	−0.087683	0.123202
2009:3	–0.06221	−0.002095	0.060086	−0.123354	0.119164
2009:4	–0.003635	0.006493	0.063164	−0.120976	0.133963
2010:1	–0.1911938	0.016435	0.065429	−0.115605	0.148475
2010:2	0.0002207	0.013870	0.066425	−0.120182	0.147922
2010:3	–0.03853	0.004754	0.067609	−0.131686	0.141194
2010:4	0.08106	0.009609	0.068220	−0.128063	0.147282
2011:1	–0.002692	0.013526	0.068644	−0.125004	0.152055
2011:2	0.1127259	0.011529	0.068840	−0.127397	0.150454
2011:3	–0.02047	0.007798	0.069059	−0.131568	0.147164
2011:4	0.08649	0.010466	0.069186	−0.129156	0.150088
2012:1	–0.04747	0.011927	0.069269	−0.127863	0.151716
2012:2	0.06716	0.010711	0.069309	−0.129160	0.150583
2012:3	–0.003477	0.009260	0.069350	−0.130694	0.149213
2012:4	0.07761	0.010650	0.069377	−0.129358	0.150658
2013:1	–0.07186	0.011142	0.069393	−0.128898	0.151182
2013:2	0.05621	0.010477	0.069401	−0.129580	0.150535
2013:3	–0.04800	0.009946	0.069409	−0.130127	0.150019
2013:4	0.06604	0.010640	0.069415	−0.129445	0.150724
2014:1	–0.09138	0.010775	0.069418	−0.129315	0.150866
2014:2	0.05304	0.010436	0.069420	−0.129658	0.150530
2014:3	–0.001114	0.010259	0.069421	−0.129838	0.150356
2014:4	0.07616	0.010592	0.069422	−0.129507	0.150692

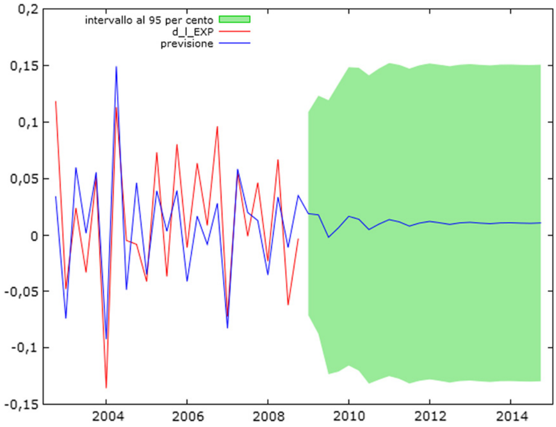

Fig. 13. Forecast for ΔEXP_t (color online)

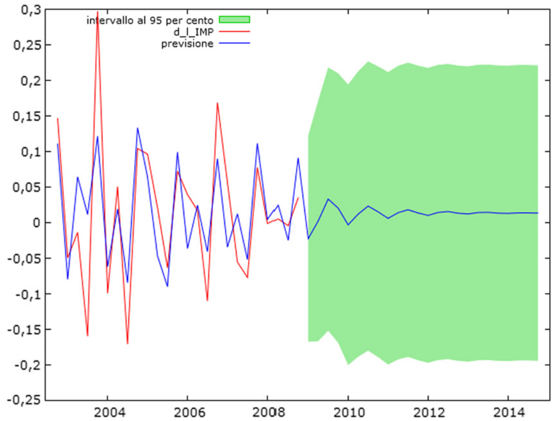

Fig. 14. Forecast for ΔIMP_t (color online)

By means of the forecasts from 2009 to 2013, we can establish a comparison with the real data, noting that the predictions with this VAR model are not very reliable since the error is quite high and it increases in recent years. The lack of accuracy was confirmed previously by low values of the *Adjusted* R^2. Figures 13, 14, and 15 show the real time series of the three variables with a red line and the prediction made with the estimated models with a blue line. It can be seen from these graphs that the confidence intervals (green area in the figures) are very high.

3.2 Second model: nonstationary variables

In this section, we analyze the three variable (EXP_t, IMP_t, and $ACTE_t$), considering quarterly Verona data from 1995 to 2013. We analyze these time series without avoiding structural breaks and without considering the first differences, and we check if the analysis produces different results with respect to the previous ones. Figure 16 shows a multiple graph for the time series respectively of EXP_t, IMP_t, and $ACTE_t$.

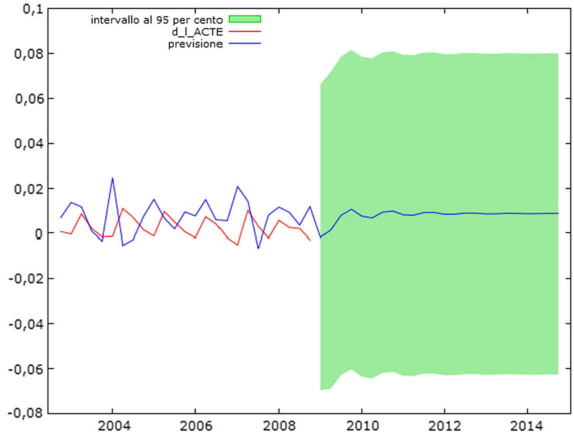

Fig. 15. Forecast for $\Delta ACTE_t$ (color online)

Table 8. VAR lag lengths

p	$AIC(p)$	$BIC(p)$
1	98.133995	98.525673
2	97.843826	98.529262
3	97.432952	98.412147*
4	97.327951	98.600904
5	97.204547	98.771258
6	97.107478	98.967947
7	97.020628	99.174856
8	97.007994*	99.455981

The GRETL lag length selection gives the results in Table 8; then, according to the considerations made to determine the number of delays for the model (7), we decide to choose three delays, obtaining the following model:

$$\begin{aligned}
EXP_t &= -119\,893\,000 + 0.79EXP_{t-1} + 0.47EXP_{t-2} - 0.31EXP_{t-3} \\
&\quad - 0.12IMP_{t-1} + 0.20IMP_{t-2} - 0.09IMP_{t-3} \\
&\quad + 4389.61ACTE_{t-1} + 4715.09ACTE_{t-2} - 6479.85ACTE_{t-3}, \\
IMP_t &= -313\,115\,000 + 0.17EXP_{t-1} + 1.11EXP_{t-2} - 1.21EXP_{t-3} \\
&\quad + 0.52IMP_{t-1} - 0.13IMP_{t-2} + 0.28IMP_{t-3} \\
&\quad + 13\,719.5ACTE_{t-1} - 3215.16ACTE_{t-2} + 1103.38ACTE_{t-3}, \\
ACTE_t &= 8526.18 - 1.62 \times 10^{-6}EXP_{t-1} + 1.87 \times 10^{-6}EXP_{t-2} \\
&\quad - 7059 \times 10^{-8}EXP_{t-3} + 1.31 \times 10^{-6}IMP_{t-1} \\
&\quad - 4.89 \times 10^{-7}IMP_{t-2} - 3.81 \times 10^{-7}IMP_{t-3} + 1.04ACTE_{t-1} \\
&\quad - 0.17ACTE_{t-2} + 0.02ACTE_{t-3}.
\end{aligned} \tag{8}$$

In the first equation (EXP_t) of VAR system (8), we have the coefficients of EXP_{t-1}, EXP_{t-2}, IMP_{t-2}, and $ACTE_{t-3}$, which are statically significant at the 1% level be-

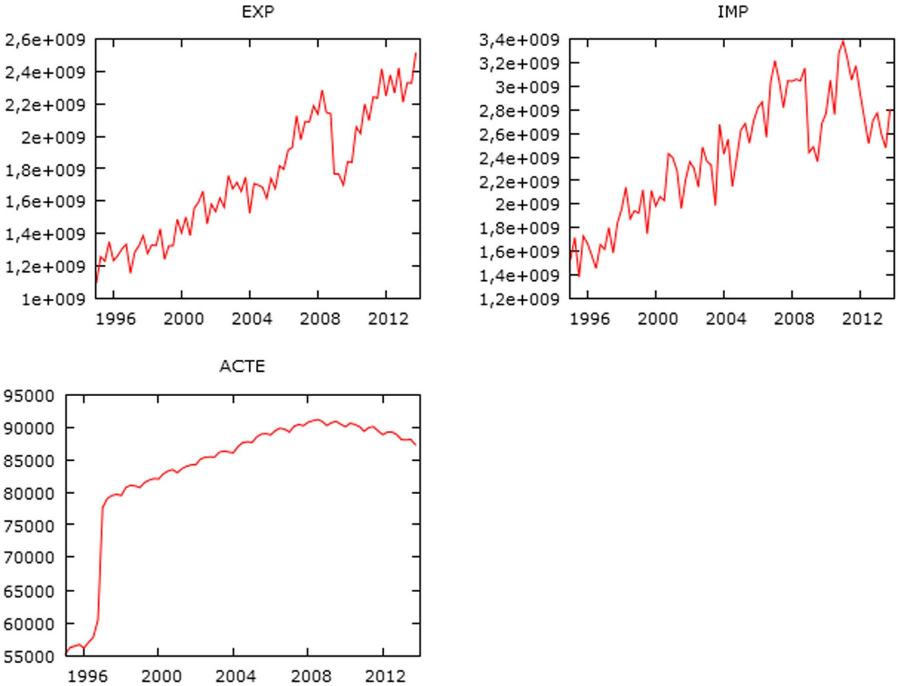

Fig. 16. Multiple Graph for EXP_t, IMP_t, and $ACTE_t$

cause their p-value is less than 0.01 and the t-statistic exceeds the critical value. However, the coefficients of EXP_{t-3} and IMP_{t-1} are statically significant at the 5% level, and the other coefficients are not statically significant. The coefficient of IMP_{t-3} is statically significant at the 10% level, and the others are not statically significant. The *Adjusted* R^2 is 0.93. In the second equation (IMP_t) of VAR system (8), we have the coefficients of EXP_{t-2}, EXP_{t-3}, IMP_{t-1}, IMP_{t-3}, and $ACTE_{t-1}$, which are statically significant at the 1% significance level. The constant is statically significant at the 5% level, and the other coefficients are not statically significant. The *Adjusted* R^2 is 0.85. In the last equation of (8), we have only EXP_{t-1}, $ACTE_{t-1}$, and $ACTE_{t-2}$ statically significant respectively at the 5%, 1%, and 10% levels, whereas the *Adjusted* R^2 is 0.94. If we perform Granger causality tests, then we have that all p-values of the F-statistic of the three equations are less than 0.01; only for the third equation of (8), the Granger causality test for the variable EXP_t has the p-value 0.0852, and hence EXP_t Granger-causes $ACTE_t$, but in this case, the null hypothesis is rejected at the level of 10%. Notice that the model (8) has high values of the *Adjusted* R^2, so it can be very useful to make prediction of future values of the three variables. The forecasts for EXP_t concerning 2014 are given by the table

Quarter	*Forecast*	*Error*
2014:I	2 415 830 000	86 744 900
2014:II	2 502 280 000	105 860 000
2014:III	2 430 160 000	143 193 000
2014:IV	2 488 770 000	158 629 000

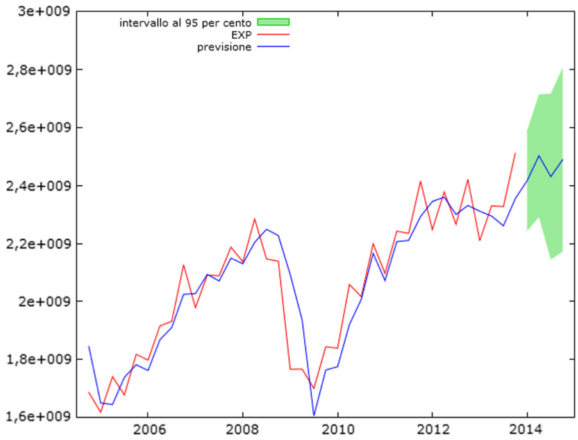

Fig. 17. Forecast for EXP_t

Forecasts for IMP_t are as follows:

Quarter	*Forecast*	*Error*
2014:I	2 764 470 000	174 343 000
2014:II	2 870 330 000	200 990 000
2014:III	2 712 960 000	237 479 000
2014:IV	2 809 610 000	249 185 000

and prediction for $ACTE_t$ reads as follows:

Quarter	*Forecast*	*Error*
2014:I	87401.59	1812.928
2014:II	87988.15	2634.586
2014:III	88266.25	3129.437
2014:IV	88495.47	3449.842

Figures 17, 18, and 19 show the time series of the three variables and their forecasts. The area of confidence interval for EXP_t is rather small, which is confirmed by the value 0.93 of the *Adjusted* R^2 of the first equation in system (8). This area is slightly wider for the second graph, and in Fig. 19 we show the confidence interval for $ACTE_t$ becoming wider at each quarter.

3.3 No cointegration between EXP_t and IMP_t

We saw in Sections 2.1 and 2.3 that the time series for EXP_t and IMP_t are both integrated of order 1 (I(1)); hence, we perform an EG-ADF test to verify if these two variables are cointegrated. The cointegrating coefficient θ is estimated by the OLS estimate of the regression $EXP_t = \alpha + \theta IMP_t + z_t$; hence, we obtain $EXP_t = 197\,119\,000 + 0.641536 IMP_t + z_t$, so that $\theta = 0.641536$. Then we use a Dickey–Fuller test to test for a unit root in $z_t = EXP_t - \theta IMP_t$. The statistic test result is –2.77065, which is greater than –3.96 (see [5, Table 1] for critical values); therefore, we cannot refuse the null hypothesis of a unit root for z_t, concluding that the series $EXP_t - \theta IMP_t$ is not stationary. Moreover, we have that the variables EXP_t and IMP_t are not cointegrated.

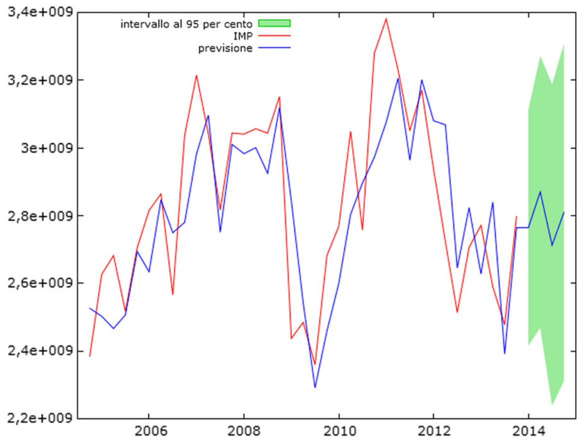

Fig. 18. Forecast for EXP_t

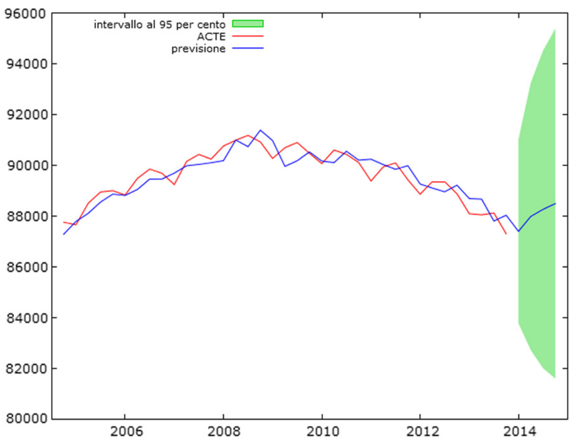

Fig. 19. Forecast for $ACTE_t$

4 VAR model with Italian data

In this section, we perform a comparison of the time series between provincial and national data. Considering the same model of system (8), but with data referring to Italy, we get a VAR(8) model of the form

$$\begin{cases} EXPn_t = \hat{\beta}_{10} + \hat{\beta}_{11} EXPn_{t-1} + \cdots + \hat{\beta}_{18} EXPn_{t-8} + \hat{\gamma}_{11} IMPn_{t-1} \\ \qquad + \cdots + \hat{\gamma}_{18} IMPn_{t-8} + \hat{\delta}_{11} ACTEn_{t-1} + \cdots + \hat{\delta}_{18} ACTEn_{t-8}, \\ IMPn_t = \hat{\beta}_{20} + \hat{\beta}_{21} EXPn_{t-1} + \cdots + \hat{\beta}_{28} EXPn_{t-8} + \hat{\gamma}_{21} IMPn_{t-1} \\ \qquad + \cdots + \hat{\gamma}_{28} IMPn_{t-8} + \hat{\delta}_{21} ACTEn_{t-1} + \cdots + \hat{\delta}_{28} ACTEn_{t-8}, \\ ACTEn_t = \hat{\beta}_{30} + \hat{\beta}_{31} EXPn_{t-1} + \cdots + \hat{\beta}_{38} EXPn_{t-8} + \hat{\gamma}_{31} IMPn_{t-1} \\ \qquad + \cdots + \hat{\gamma}_{38} IMPn_{t-8} + \hat{\delta}_{31} ACTEn_{t-1} + \cdots + \hat{\delta}_{38} ACTEn_{t-8}, \end{cases} \tag{9}$$

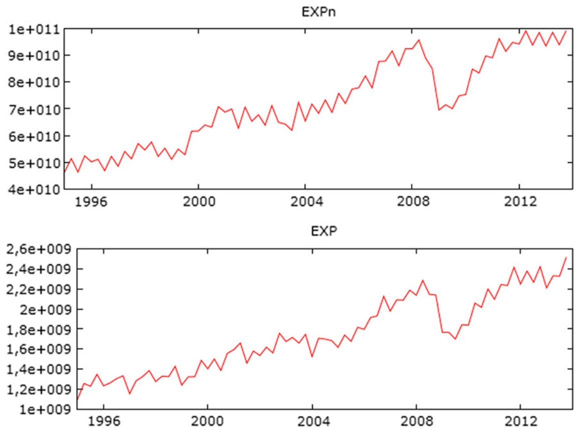

Fig. 20. Comparison between $EXPn_t$ and EXP_t

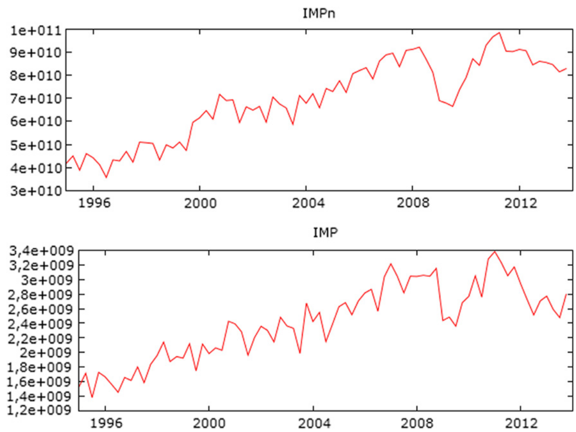

Fig. 21. Comparison of $IMPn_t$ and IMP_t

where the letter n in the variable name indicates that we are working with national data.

The *Adjusted* R^2 of the three equations in system (9) are respectively 0.95, 0.96, and 0.98. So this is a good VAR model; in fact, Granger causality tests for (9) present all p-values of the F-statistic less than 0.01. So all the three variables can be used to explain the others. In Figs. 20, 21, and 22, we note the extreme similarity of the provincial and national time series. If we perform an EG-ADF test to verify if this three couples of variables are cointegrated, then we obtain that only the variables $ACTEn_t$ and $ACTE_t$ are cointegrated with cointegrating coefficient $\theta = 49.4948$. By comparing the correlation between a variable of national data and the corresponding variables with provincial data we note a high correlation level, even taking into account the provincial variable delays. Below we present the correlation between *EXPn*

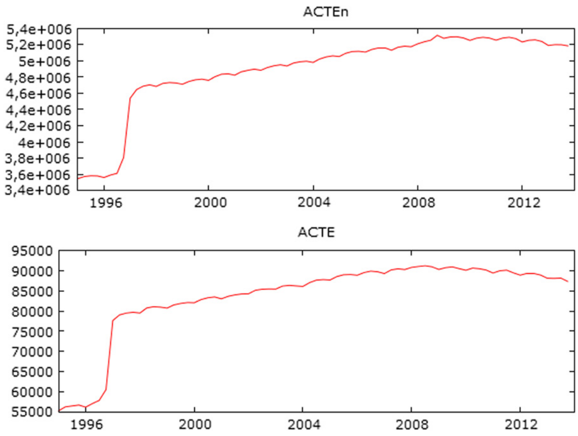

Fig. 22. Comparison of $ACTEn_t$ and $ACTE_t$

and the delays of *EXP*:

p	$corr(EXPn_t; EXP_{t+p})$
–4	0.7918
–3	0.8083
–2	0.8985
–1	0.9036
0	0.9823
1	0.8880
2	0.8677
3	0.7711
4	0.7557

Then we have the correlation between *IMPn* and the delays of *IMP*

p	$corr(IMPn_t; IMP_{t+p})$
–4	0.7490
–3	0.7645
–2	0.8428
–1	0.8745
0	0.9641
1	0.8780
2	0.8518
3	0.7887
4	0.7948

whereas the correlation between *ACTEn* and the delays of *ACTE* are given by

p	$corr(IMPn_t; IMP_{t+p})$
–4	0.6493
–3	0.7400
–2	0.8290
–1	0.9162
0	0.9947
1	0.9257
2	0.8464
3	0.7634
4	0.6771

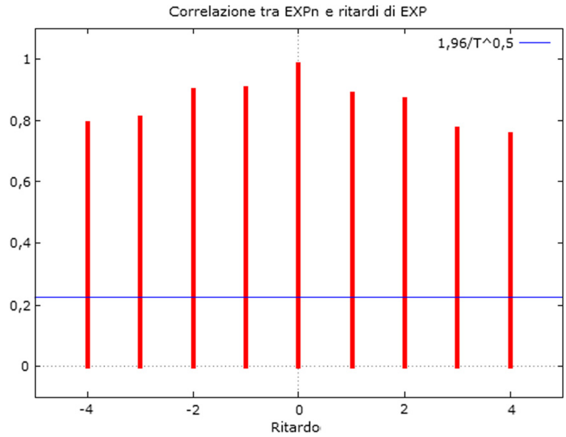

Fig. 23. Correlation between *EXPn* and *EXP*

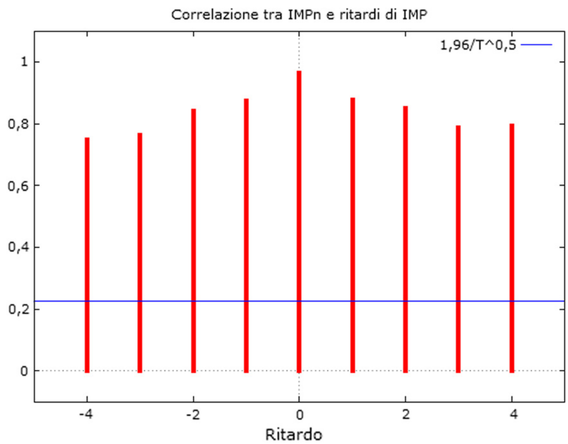

Fig. 24. Correlation between *IMPn* and *IMP*

Figures 23, 24, and 25 show the correlation diagram related to the national and provincial variables. We notice very high values, which show the strong connection between what happens at the national and the provincial levels.

5 Conclusion

We have presented an analysis of relevant time series related to the import and export data concerning the Province of Verona, together with a forecast analysis of the 2014 trend. Exploited techniques have been treated in our first paper, and these two articles together constitute a unitary project. In this second part, we have paid attention to the quantitative influence that certain macro economical events may have on considered time series. In particular, we extrapolated three particularly significant moments, namely the 2007–2008 world financial economic crisis, with consequent decrease of

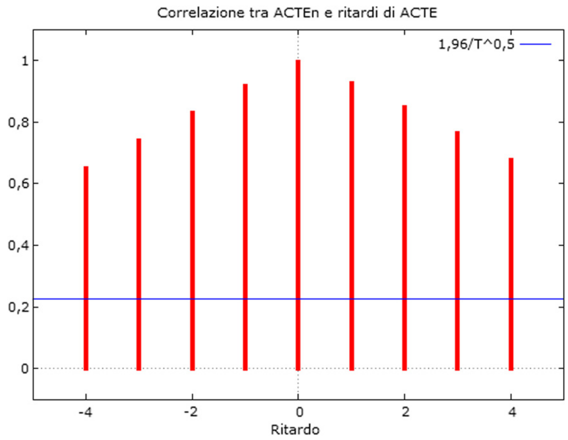

Fig. 25. Correlation between *ACTEn* and *ACTE*

import–export, a break in 1995 probably due to the devaluation of the Lira, which did not cause a decrease of the import, but resulted in an increase in exports of Verona, and the vertical growth of the *Active enterprises* parameter during 1995–1998, which has been caused by a change in the related provincial regulation. It is worth to underline how our analysis shows, by obtained numerical forecasts, a concrete possibility for a partial recovery from the present economic crisis, especially when taking into account the first quarters of 2014 and particularly with regard to exports. The results obtained can be used for concrete actions aimed, for example, to the optimization of territory economic resources, even if a concrete economical program needs of a deeper treatment for which, however, our analysis constitutes a rigorous and effective basis. Concerning the latter, possible extensions may be focused on analyzing import and export time series of specific products to underline in which areas Verona is more specialized; then such results could be used to understand where to invest more. Moreover, we could perform a comparison analysis with analogous data belonging to other cities of similar economical size, both in Italy and within the European Community.

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