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Corrigendum to "A new confidence interval based on the theory of U-statistics for the area under the curve"[1]

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1 Issues

We noted three issues about our article "A new confidence interval based on the theory of U-statistics for the area under the curve", Modern Stochastics: Theory and Applications 13 (2026), 57–82:

1. The proof of Lemma 3 is not correct. We have not thought about the finiteness of the integral

$$\int_{\{v\in\mathbb{R}^p\,|\,\beta^T v=0\}} \frac{\langle v,u\rangle}{\|\beta\|} \int_{\mathbb{R}^p} q(w)\cdot\pi(\beta_0^T w)\cdot q(v+w)\cdot(1-\pi(\beta_0^T(v+w)))\,dw\,dv$$

appearing on page 76, line 2. However, if both the positive part and the negative part of that integral are infinite, the proof of Lemma 3 breaks down. We are not sure whether Lemma 3 holds under the regularity assumptions made in the article. Below we present a proof using different regularity assumptions.

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2. On page 79, line 14/15, the central limit theorem for the maximum likelihood estimator $\hat{\beta}$ of a logistic regression model from [2] is applied. However, in [2] deterministic design points are considered, while in our article random design points are considered. So this theorem cannot be applied.

3. It is assumed that the design points are absolutely continuous and a certain functional of the design points has a bounded density. With some modest change of the method of the proofs, this assumption can be relaxed to saying that the probability of the design points lying in a hyperplane is zero for certain hyperplanes. Notice that this does not mean that the results of the paper are wrong, but only that they can be improved with little effort.

2 Corrections

The definition of Model 2 has to be modified as follows:

Model 2. *Let $\hat{Y}_{2,i}, i = 1, \ldots, n$, be the fitted values of a logistic regression model (see Section 2) with true parameter $\beta_0 \in \mathbb{R}^p$, applied to i.i.d. data points $X_{2,i}, i = 1, \ldots, n$, and let $I_{2,i}$, $1, \ldots, n$, be the known true classification. We assume $\beta_0 \neq 0$. We shall assume that the design points have finite second moment, $\mathbb{E}\,\|X_{2,i}\|^2 < \infty$, $i = 1, \ldots, n$, and that $\mathbb{P}(X_{2,i} \in H) = 0$ for every affine hyperplane $H \subseteq \mathbb{R}^p$ perpendicular to β_0 and for every linear hyperplane $H \subseteq \mathbb{R}^p$ (no matter whether it is perpendicular to β_0 or not). Moreover, the cardinality m of the training set and the cardinality n of the test set should fulfill*

$$\liminf_{n\to\infty} \frac{m}{n} > 0, \ \limsup_{n\to\infty} \frac{m}{n} < \infty. \tag{1}$$

Lemma 3 and its proof have to be modified as follows:

Lemma 3. *Put*

$$z(\beta) := \mathbb{P}(\beta^T X_1 < \beta^T X_2, I_1 = 0, I_2 = 1) + \frac{1}{2}\mathbb{P}(\beta^T X_1 = \beta^T X_2, I_1 = 0, I_2 = 1).$$

Then, under Model 2, z is differentiable in β_0 with

$$\frac{d}{d\beta} z(\beta)_{|\beta=\beta_0}.$$

Notice, first, that the second summand in the definition of z has to be mentioned explicitely now, since while $\mathbb{P}(\beta_0^T X_1 = \beta_0^T X_2, I_1 = 0, I_2 = 1) = 0$ still holds, it is not sure that $\mathbb{P}(\beta^T X_1 = \beta^T X_2, I_1 = 0, I_2 = 1) = 0$ holds for $\beta \neq \beta_0$ any more. Notice, moreover, that we do not get differentiability (not even continuity) for $\beta \neq \beta_0$ any more.

Proof. Observe

$$z(\beta) := \frac{1}{2}\mathbb{P}(\beta^T X_1 \leq \beta^T X_2, I_1 = 0, I_2 = 1) + \frac{1}{2}\mathbb{P}(\beta^T X_1 > \beta^T X_2, I_1 = 1, I_2 = 0),$$

since $(X_1, I_1, X_2, I_2) = (X_2, I_2, X_1, I_1)$ in distribution.

Let $(\beta_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^p \setminus \{\beta_0\}$ converging to β_0.
Recall $\pi(t) = \frac{1}{1+\exp\{t\}}$ so that $\mathbb{P}(I_1 = 0 \mid X_1) = \pi(\beta_0^T X_1)$. Then

$$\begin{aligned}
z(\beta_n) - z(\beta_0) =& \mathbb{E}\Big[\frac{1}{2}\mathbb{P}(\beta_n^T X_1 \le \beta_n^T X_2, I_1 = 0, I_2 = 1 \mid X_1, X_2) \\
&+\frac{1}{2}\mathbb{P}(\beta_n^T X_1 > \beta_n^T X_2, I_1 = 1, I_2 = 0 \mid X_1, X_2) \\
&-\frac{1}{2}\mathbb{P}(\beta_0^T X_1 \le \beta_0^T X_2, I_1 = 0, I_2 = 1 \mid X_1, X_2) \\
&-\frac{1}{2}\mathbb{P}(\beta_0^T X_1 > \beta_0^T X_2, I_1 = 1, I_2 = 0 \mid X_1, X_2)\Big] \\
=& \frac{1}{2}\int_{\mathbb{R}^p}\int_{\mathbb{R}^p} \mathbf{1}_{\{\beta_n^T x_1 \le \beta_n^T x_2\}} \cdot \pi(\beta_0^T x_1) \cdot (1 - \pi(\beta_0^T x_2)) \\
&+\mathbf{1}_{\{\beta_n^T x_1 > \beta_n^T x_2\}} \cdot (1 - \pi(\beta_0^T x_1)) \cdot \pi(\beta_0^T x_2) \\
&-\mathbf{1}_{\{\beta_0^T x_1 \le \beta_0^T x_2\}} \cdot \pi(\beta_0^T x_1) \cdot (1 - \pi(\beta_0^T x_2)) \\
&+\mathbf{1}_{\{\beta_0^T x_1 > \beta_0^T x_2\}} \cdot (1 - \pi(\beta_0^T x_1)) \cdot \pi(\beta_0^T x_2)\, d\mu(x_1)\, d\mu(x_2).
\end{aligned}$$

Obviously, the integrand is zero outside the symmetric difference Δ_n of $\{\beta_n^T x_1 \le \beta_n^T x_2\}$ and $\{\beta_0^T x_1 \le \beta_0^T x_2\}$. It is also zero on the set $\Delta_= := \{\beta_0^T x_1 = \beta_0^T x_2\}$. On the whole of $\mathbb{R}^p \times \mathbb{R}^p$, the absolute value of the integrand can be bounded from above by

$$\begin{aligned}
&|\pi(\beta_0^T x_1) \cdot (1 - \pi(\beta_0^T x_2)) - \pi(\beta_0^T x_2) \cdot (1 - \pi(\beta_0^T x_1))| \\
\le& |\pi(\beta_0^T x_1) - \pi(\beta_0^T x_2)| + |(1 - \pi(\beta_0^T x_2)) - (1 - \pi(\beta_0^T x_1))| \\
\le& 2 \cdot 1/4 \cdot |\beta_0^T x_1 - \beta_0^T x_2|,
\end{aligned}$$

since π is Lipschitz continuous with Lipschitz constant $\frac{1}{4}$.

We must have $|\beta_0^T(x_1 - x_2)| \le |(\beta_n - \beta_0)^T(x_1 - x_2)|$ on Δ_n, since $|\beta_0^T(x_1 - x_2)| > |(\beta_n - \beta_0)^T(x_1 - x_2)|$ implies that $\beta_0^T(x_1 - x_2)$ and $\beta_n^T(x_1 - x_2)$ have the same sign and thus $(x_1, x_2) \notin \Delta_n$. Using that the Cauchy-Schwarz inequality gives $|(\beta_n - \beta_0)^T(x_1 - x_2)| \le \|\beta_n - \beta_0\| \cdot \|x_1 - x_2\|$, we get

$$|z(\beta_n) - z(\beta_0)| \le \frac{1}{4}\|\beta_n - \beta_0\| \cdot \int_{\mathbb{R}^p}\int_{\mathbb{R}^p} \mathbf{1}_{\{(x_1,x_2)\in\Delta_n\setminus\Delta_=\}}\|x_1 - x_2\|\, d\mu(x_1)\, d\mu(x_2).$$

Now $\lim_{n\to\infty} \mathbf{1}_{\{(x_1,x_2)\in\Delta_n\setminus\Delta_=\}} = 0$ for any $x_1, x_2 \in \mathbb{R}^p$ implies

$$\lim_{n\to\infty} \frac{|z(\beta_n) - z(\beta_0)|}{\|\beta_n - \beta_0\|} \le \frac{1}{4} \lim_{n\to\infty} \int_{\mathbb{R}^p}\int_{\mathbb{R}^p} \mathbf{1}_{\{(x_1,x_2)\in\Delta_n\setminus\Delta_=\}}\|x_1 - x_2\|\, d\mu(x_1)\, d\mu(x_2) = 0,$$

since we are allowed to apply the dominated convergence theorem due to

$$\begin{aligned}
&\int_{\mathbb{R}^p}\int_{\mathbb{R}^p} \mathbf{1}_{\{(x_1,x_2)\in\Delta_n\setminus\Delta_=\}}\|x_1 - x_2\|\, d\mu(x_1)\, d\mu(x_2) \\
&\le \int_{\mathbb{R}^p} \|x_1\|\, d\mu(x_1) + \int_{\mathbb{R}^p} \|x_2\|\, d\mu(x_2) < \infty.
\end{aligned}$$

This implies that z is differentiable in β_0 with derivative 0. □

Equality (4) on page 79 has now to be derived in a different way. Put

$$\bar{D}(\beta) = \{x \in S^{p-1} \mid \beta^T x \leq 0, \beta_0^T x \geq 0\} \cup \{x \in S^{p-1} \mid \beta^T x \geq 0, \beta_0^T x \leq 0\}$$

and

$$D_= = \{x \in S^{p-1} \mid \beta_0^T x = 0\}.$$

Then the strong consistency of $\hat{\beta}$ (Corollary 3 of [1]) implies $\bigcap_{n\in\mathbb{N}} \bigcup_{N\geq n} \bar{D}(\hat{\beta}_N) \subseteq D_=$ a.s. Hence

$$\begin{aligned}
&\mathbb{P}(\sqrt{n} \cdot |U_1(\hat{\beta}) - \mathbb{E}[U_1(\hat{\beta}) \mid \hat{\beta}] - U_1(\beta_0) + \mathbb{E}U_1(\beta_0)| > \epsilon) \\
&\quad = \mathbb{E}\big[\mathbb{P}(\sqrt{n} \cdot |U_1(\hat{\beta}) - \mathbb{E}[U_1(\hat{\beta}) \mid \hat{\beta}] - U_1(\beta_0) + \mathbb{E}U_1(\beta_0)| > \epsilon \mid \hat{\beta})\big] \\
&\quad \leq \mathbb{E}\Big[\frac{8}{\epsilon^2} \cdot \mathbb{P}\Big(\frac{X_2 - X_1}{\|X_2 - X_1\|} \in \bar{D}(\hat{\beta}) \mid \hat{\beta}\Big) + \frac{8}{n\epsilon^2}\Big] \\
&\quad \leq \mathbb{E}\Big[\frac{8}{\epsilon^2} \cdot \mathbb{P}\Big(\frac{X_2 - X_1}{\|X_2 - X_1\|} \in \bigcup_{N\geq n} \bar{D}(\hat{\beta}_N) \cup D_= \mid \hat{\beta}\Big) + \frac{8}{n\epsilon^2}\Big] \\
&\quad \longrightarrow \mathbb{E}\Big[\frac{8}{\epsilon^2} \cdot \mathbb{P}\Big(\frac{X_2 - X_1}{\|X_2 - X_1\|} \in D_= \mid \hat{\beta}\Big)\Big] \\
&\quad = \frac{8}{\epsilon^2} \cdot \mathbb{P}(\beta_0^T X_1 = \beta_0^T X_2) \\
&\quad = 0,
\end{aligned}$$

where the first inequality is derived by arguments of page 78 of the article.

On page 79, line 14/15, the central limit theorem of [2] cannot be used. Instead one has to use Corollary 3 of [1].

References

[1] Fahrmeir, L., Kaufmann, H.: Consistency and asymptotic normality of the maximum likelihood estimator in generalized linear models. Ann. Stat. **13**, 342–368 (1985) MR0773172. https://doi.org/10.1214/aos/1176346597

[2] Fahrmeir, L., Kneib, T., Lang, S.: Regression – Modelle, Methoden und Anwendungen. Springer (2007)