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Strong uniqueness of solutions of stochastic differential equations with jumps and non-Lipschitz random coefficients

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Abstract In the paper we establish strong uniqueness of solution of a system of stochastic differential equations with random non-Lipschitz coefficients that involve both the square integrable continuous vector martingales and centered and non-centered Poisson measures.

Keywords Stochastic differential equations, non-Lipschitz coefficients, Poisson measure, uniqueness of a solution
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1 Introduction

Let $(\Omega, \Im, \Im_t, t \geq 0, P)$ be a complete probability space with filtration. Let $\{\zeta_k(t),\ k = \overline{1, m}\}$ be continuous square integrable martingales defined on the stochastic basis $(\Omega, \Im, \Im_t, t \geq 0, P)$ with square characteristics $\langle \zeta_k \rangle$ satisfying the following relations of orthogonality:

$$E([\zeta_k(t) - \zeta_k(s)][\zeta_i(t) - \zeta_i(s)]/\Im_s) = \begin{cases} E([\langle \zeta_k \rangle(t) - \langle \zeta_k \rangle(s)]/\Im_s), & k = i, \\ 0, & k \neq i \end{cases}$$

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with probability 1 for any $s < t$. Let $(\Theta, \mathcal{B}_\Theta)$ be a measurable space admitting some expansion $\Theta = \Theta_1 \cup \Theta_2$ with $\Theta_1 \cap \Theta_2 = \emptyset$. Also, let $\nu([0,t],A)$ be a Poisson measure with the parameter $t\Pi(A)$, $A \in \mathcal{B}_\Theta$ and let $\tilde{\nu}([0,t],A) = \nu([0,t],A) - t\Pi(A)$. Suppose that $\Pi(\Theta_1) = \infty$, $\int_{\Theta_1} |\theta|^2 \Pi(d\theta) < \infty$, $\Pi(\Theta_2) < \infty$ and $\nu([0,t],A)$ are $\Im_t$-measurable. Also suppose that the family of random variables $\nu([t,t+h],A)$, $h > 0$ is independent on $\Im_t$ and that the processes $\{\zeta_k(t),\ k = \overline{1,m}\}$ are jointly independent with the measure $\nu([0,t],A)$. Consider random vector-valued functions $a(t,x)$, $b_k(t,x)$, $f_i(t,x,\theta)$ with the values in $\mathbb{R}^m$, $x \in \mathbb{R}^m$, $\theta \in \Theta$ and suppose that they are $\Im_t$-measurable and continuous with probability 1 on the aggregate variables t, x, θ. At last, suppose that $\xi(0)$ is a given random variable that is $\Im_0$-measurable.

In the present paper we generalize the result of Kulinich [4] on a strong uniqueness of the solution ξ for the following system of stochastic differential equations

$$d\xi(t) = a(t,\xi(t))\,dt + \sum_{k=1}^{m} b_k(t,\xi(t))\,d\zeta_k(t) + \int_{\Theta_1} f_1(t,\xi(t-),\theta)\,\tilde{\nu}(dt,d\theta)$$
$$+ \int_{\Theta_2} f_2(t,\xi(t-),\theta)\,\nu(dt,d\theta), \quad t > 0. \tag{1}$$

A strong solution of equation (1) is $\Im_t$-progressively measurable stochastic process $\xi(t) = (\xi_i(t),\ i = \overline{1,m})$, which is continuous on the right and without discontinuities of the second kind and whose stochastic differential takes form (1) (see [2]).

In the paper [4] the system of stochastic differential equations (1) was considered only with $f_2(t,x,\theta) \equiv 0$ and nonrandom coefficients under additional condition of boundedness of the functions $a(t,x)$, $b_k(t,x)$, $\int_{\Theta_1} |f_1(t,x,\theta)|^l \Pi(d\theta)$, $l = 1,2$. In the present paper these restrictions are not required.

The problem of a strong uniqueness of a solution of the equation (1) with Lipschitz coefficients was considered in a number of works, among which we mention only monograph [2] that contains both an extended bibliography and analysis of the state of art. The problem of a strong uniqueness of the solution in the case of non-Lipschitz coefficients was examined in a small number of papers and only for one-dimensional equations. We point out the papers [5, 6, 8], in which the new original methods of investigation of a strong uniqueness of solution for one-dimensional equations of form (1) without jumps and with $\zeta_k(t) = W(t)$ are proposed. Then in [3, 7, 9] these methods were extended to wider classes of one-dimensional equations of form (1) but also with $\zeta_k(t) = W(t)$.

The most complete study of the problem under consideration is presented in [2], but there the conditions on the smoothness of the coefficients $b_k(t,x)$ in the multidimensional case are stronger than in our paper. However, it is important to note that we can significantly weaken the conditions on the coefficients $b_k(t,x)$ of equation (1) only for a special class of equations, in particular, for the equations containing Hölder's coefficients with the index $\alpha \geqslant \frac{1}{2}$. For example, the coefficients $b_1(t,x) = (t\sqrt{|x_1|}, \sin t)$, $b_2(t,x) = (\cos t, \sqrt{|x_2|})$ satisfy conditions of our theorem but were inappropriate for any previous theorems. We apply the method proposed in [8] to prove the main result.

2 Strong uniqueness

Now we state and prove the main result.

Theorem. *Let for all $N > 0$ functions $q_N(r)$, $\rho_N(r)$, $r \geqslant 0$ are nondecreasing and continuous and additionally $q_N(r)$ is concave. Suppose that for all $t \leqslant N$, $|x| \leqslant N$, $|y| \leqslant N$ the following inequalities hold true with probability* 1

$$|a(t,x) - a(t,y)| + \int_{\Theta_1} |f_1(t,x,\theta) - f_1(t,y,\theta)| \,\Pi(d\theta) \leqslant q_N(|x-y|),$$
$$|b_{ik}(t,x) - b_{ik}(t,y)| \leqslant \rho_N(|x_i - y_i|), \quad i = \overline{1,m},\ k = \overline{1,m}.$$

Also let

$$\int_{0+} \frac{dr}{q_N(r)} = \int_{0+} \frac{dr}{\rho_N^2(r)} = \infty. \tag{2}$$

Then the strong solution ξ of equation (1) is strongly unique.

Proof. It is known (see [2]) that in order to prove strong uniqueness of the solution of equation (1), it is enough to establish strong uniqueness of its solution in every random interval $[\tau_l, \tau_{l+1}]$, where $0 = \tau_0 < \tau_1 < \tau_2 < \cdots$ are subsequent jump times of Poisson process $\nu([0,t],\Theta_2)$, assuming additionally that $\xi(\tau_l)$ is given.

Now, let $\xi(t)$ and $\tilde{\xi}(t)$ be two solutions of equation (1) with the same initial condition. Then for $\tau_l \leqslant t < \tau_{l+1}$ we have that $\nu((\tau_l,\tau_{l+1}),\Theta_2) = 0$, therefore

$$\xi_i(t) = \xi_i(\tau_l) + \int_{\tau_l}^t a_i(s,\xi(s))\,ds + \sum_{k=1}^m \int_{\tau_l}^t b_{ik}(s,\xi(s))\,d\zeta_k(s)$$
$$+ \int_{\tau_l}^t \int_{\Theta_1} f_{i1}(s,\xi(s-),\theta)\,\tilde{\nu}(ds,d\theta),$$

$i = \overline{1,m}$. A similar equation we have for the process $\tilde{\xi}(t)$.

So,

$$\Delta\xi_i(t) = \int_{\tau_l}^t \Delta a_i(s)\,ds + \sum_{k=1}^m \int_{\tau_l}^t \Delta b_{ik}(s)\,d\zeta_k(s)$$
$$+ \int_{\tau_l}^t \int_{\Theta_1} \Delta f_{i1}(s,\theta)\,\tilde{\nu}(ds,d\theta), \tag{3}$$

$i = \overline{1,m}$, where

$$\Delta\xi_i(t) = \xi_i(t) - \tilde{\xi}_i(t),$$
$$\Delta a_i(t) = a_i(t,\xi(t)) - a_i(t,\tilde{\xi}(t)),$$
$$\Delta b_{ik}(t) = b_{ik}(t,\xi(t)) - b_{ik}(t,\tilde{\xi}(t)),$$
$$\Delta f_{i1}(t,\theta) = f_{i1}(t,\xi(t-),\theta) - f_{i1}(t,\tilde{\xi}(t-),\theta).$$

Then we use the method that was firstly proposed in [8]. According to conditions (2) on the function $\rho_N(r)$, we can choose a monotone sequence

$$1 = a_0 > a_1 > \cdots > a_n \to 0 \quad \text{as } n \to \infty$$

such that

$$\int_{a_n}^{a_{n-1}} \rho_N^{-2}(r)\,dr = n, \quad n = 1,2,\ldots$$

Let us construct a sequence of twice continuously differentiable functions $\varphi_n(r)$, $r \in [0,\infty)$ such that $\varphi_n(0) = 0$; $\varphi_n'(r) = 0$ for $0 \leqslant r \leqslant a_n$, the values $\varphi_n'(r)$ lie between 0 and 1 for $a_n < r < a_{n-1}$, $\varphi_n'(r) = 1$ for $r \geqslant a_{n-1}$; $\varphi_n''(r) = 0$ for $0 \leqslant r \leqslant a_n$, the values $\varphi_n''(r)$ lie between 0 and $\frac{2}{n}\rho_N^{-2}(r)$ for $a_n < r < a_{n-1}$, $\varphi_n''(r) = 0$ for $r \geqslant a_{n-1}$. Continuing $\varphi_n(r)$ in a symmetric way to $(-\infty,0)$, we obtain the sequence of twice continuously differentiable functions $g_n(z) = \varphi_n(|z|)$, moreover $g_n(z) \uparrow |z|$ as $n \to \infty$.

From equality (3) and Itô's formula (see [2]) for $\tau_l \leqslant t < \tau_{l+1}$ we get

$$\begin{aligned} g_n(\Delta\xi_i(t)) &= \int_{\tau_l}^t g_n'(\Delta\xi_i(s))\,\Delta a_i(s)\,ds \\ &\quad + \frac{1}{2}\sum_{k=1}^m \int_{\tau_l}^t g_n''(\Delta\xi_i(s))\,[\Delta b_{ik}(s)]^2\,d\langle\zeta_k\rangle(s) \\ &\quad + \int_{\tau_l}^t \int_{\Theta_1} [g_n(\Delta\xi_i(s) + \Delta f_{i1}(s,\theta)) - g_n(\Delta\xi_i(s)) \\ &\quad - g_n'(\Delta\xi_i(s))\,\Delta f_{i1}(s,\theta)]\,\Pi(d\theta)\,ds \\ &\quad + \sum_{k=1}^m \int_{\tau_l}^t g_n'(\Delta\xi_i(s))\,\Delta b_{ik}(s)\,d\zeta_k(s) \\ &\quad + \int_{\tau_l}^t \int_{\Theta_1} [g_n(\Delta\xi_i(s-) + \Delta f_{i1}(s,\theta)) - g_n(\Delta\xi_i(s-))]\,\tilde{\nu}(ds,d\theta) \\ &= I_1(t) + \cdots + I_5(t), \quad i = \overline{1,m}. \end{aligned} \tag{4}$$

Let us consider the stopping times $\zeta_N = \inf\{t > \tau_l : |\xi(t)| + |\tilde{\xi}(t)| > N\} \wedge \tau_{l+1} \wedge N$. Taking into account (4) and the obvious relation $g_n(z) \geqslant 0$ we get the inequalities

$$\begin{aligned} g_n(\Delta\xi_i(t))\,\chi_{\{t<\zeta_N\}} &= \sum_{k=1}^5 I_k(t)\,\chi_{\{t<\zeta_N\}} \\ &= \Big(\sum_{k=1}^5 \hat{I}_k(t)\Big)\chi_{\{t<\zeta_N\}} \leqslant \sum_{k=1}^5 \hat{I}_k(t) \quad \text{a.s.,} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \hat{I}_1(t) &= \int_{\tau_l}^t g_n'(\Delta\xi_i(s))\,\Delta a_i(s)\,\chi_{\{s<\zeta_N\}}\,ds, \\ \hat{I}_2(t) &= \frac{1}{2}\sum_{k=1}^m \int_{\tau_l}^t g_n''(\Delta\xi_i(s))\,[\Delta b_{ik}(s)]^2\,\chi_{\{s<\zeta_N\}}\,d\langle\zeta_k\rangle(s), \\ \hat{I}_3(t) &= \int_{\tau_l}^t \int_{\Theta_1} [g_n(\Delta\xi_i(s) + \Delta f_{i1}(s,\theta)) - g_n(\Delta\xi_i(s)) \\ &\quad - g_n'(\Delta\xi_i(s))\,\Delta f_{i1}(s,\theta)]\,\chi_{\{s<\zeta_N\}}\,\Pi(d\theta)\,ds, \\ \hat{I}_4(t) &= \sum_{k=1}^m \int_{\tau_l}^t g_n'(\Delta\xi_i(s))\,\Delta b_{ik}(s)\,\chi_{\{s<\zeta_N\}}\,d\zeta_k(s), \\ \hat{I}_5(t) &= \int_{\tau_l}^t \int_{\Theta_1} [g_n(\Delta\xi_i(s-) + \Delta f_{i1}(s,\theta)) - g_n(\Delta\xi_i(s-))]\,\chi_{\{s<\zeta_N\}}\,\tilde{\nu}(ds,d\theta). \end{aligned}$$

Continuity of the functions $q_N(r)$, $\rho_N(r)$ and standard properties of stochastic integrals lead to equalities

$$E(\hat{I}_4(t)/\Im_{\tau_l}) = 0, \qquad E(\hat{I}_5(t)/\Im_{\tau_l}) = 0$$

with probability 1.

Therefore

$$E(g_n(\Delta\xi_i(t))\chi_{\{t<\zeta_N\}}/\Im_{\tau_l}) \leqslant \sum_{k=1}^{3} E(\hat{I}_k(t)/\Im_{\tau_l}). \tag{6}$$

According to the assumptions of the theorem, we have

$$\begin{aligned}
&|E(\hat{I}_1(t)/\Im_{\tau_l})| \\
&\quad \leqslant E\Big(\int_{\tau_l}^{t} g_n'(\Delta\xi_i(s))\,|\Delta a_i(s)|\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big) \\
&\quad \leqslant E\Big(\int_{\tau_l}^{t} q_N(|\Delta\xi(s)|)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big); \\
&|E(\hat{I}_2(t)/\Im_{\tau_l})| \\
&\quad \leqslant \frac{1}{2}\sum_{k=1}^{m} E\Big(\int_{\tau_l}^{t} \frac{2}{n\rho_N^2(|\Delta\xi_i(s)|)} \\
&\qquad \times \chi_{\{a_n<|\Delta\xi_i(s)|<a_{n-1}\}}(\Delta b_{ik}(s))^2\chi_{\{s<\zeta_N\}}\,d\langle\zeta_k\rangle(s)/\Im_{\tau_l}\Big) \\
&\quad \leqslant \frac{1}{n}\sum_{k=1}^{m} E\Big(\int_{\tau_l}^{t} \frac{1}{\rho_N^2(|\Delta\xi_i(s)|)} \\
&\qquad \times \chi_{\{a_n<|\Delta\xi_i(s)|<a_{n-1}\}}\rho_N^2(|\Delta\xi_i(s)|)\chi_{\{s<\zeta_N\}}\,d\langle\zeta_k\rangle(s)/\Im_{\tau_l}\Big) \\
&\quad \leqslant \frac{1}{n}\sum_{k=1}^{m} E\Big(\int_{\tau_l}^{t} \chi_{\{s<\zeta_N\}}\,d\langle\zeta_k\rangle(s)/\Im_{\tau_l}\Big) \\
&\quad = \frac{1}{n}\sum_{k=1}^{m} E([\langle\zeta_k\rangle(t\wedge\zeta_N) - \langle\zeta_k\rangle(\tau_l\wedge\zeta_N)]/\Im_{\tau_l}) \to 0 \quad \text{as } n\to\infty; \\
&|E(\hat{I}_3(t)/\Im_{\tau_l})| \\
&\quad \leqslant E(\int_{\tau_l}^{t}\int_{\Theta_1} [|g_n'(\Delta\xi_i(s) + \tilde{\theta}\Delta f_{i1}(s,\theta))||\Delta f_{i1}(s,\theta)| \\
&\qquad + |g_n'(\Delta\xi_i(s))||\Delta f_{i1}(s,\theta)|]\,\Pi(d\theta)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}) \\
&\quad \leqslant 2E\Big(\int_{\tau_l}^{t} q_N(|\Delta\xi(s)|)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big).
\end{aligned}$$

Therefore

$$E(g_n(\Delta\xi_i(t))\chi_{\{t<\zeta_N\}}/\Im_{\tau_l}) \leqslant 3E\Big(\int_{\tau_l}^{t} q_N(|\Delta\xi(s)|)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big) + o_P(1),$$

where $o_P(1)\to 0$ as $n\to\infty$ in probability.

From the convergence $g_n(z) \uparrow |z|$ as $n \to \infty$ and the previous inequality we obtain

$$E(|\Delta\xi_i(t)|\chi_{\{t<\zeta_N\}}/\Im_{\tau_l}) \leqslant 3E\Big(\int_{\tau_l}^t q_N(|\Delta\xi(s)|)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big)$$

for all $i = \overline{1,m}$. Therefore

$$E(|\Delta\xi(t)|\chi_{\{t<\zeta_N\}}/\Im_{\tau_l}) \leqslant CE\Big(\int_{\tau_l}^t q_N(|\Delta\xi(s)|)\,\chi_{\{s<\zeta_N\}}\,ds/\Im_{\tau_l}\Big)$$

and

$$E(|\Delta\xi(t)|\chi_{\{t<\zeta_N\}}/\Im_{\tau_l}) \leqslant CE\Big(\int_{\tau_l}^t q_N(|\Delta\xi(s)|\,\chi_{\{s<\zeta_N\}})\,ds/\Im_{\tau_l}\Big).$$

This inequality holds true if t is replaced by the stopping time $\eta_u = u + \tau_l$. Therefore

$$\begin{aligned}E(|\Delta\xi(\eta_u)|\chi_{\{\eta_u<\zeta_N\}}/\Im_{\tau_l}) &\leqslant CE\Big(\int_{\tau_l}^{\eta_u} q_N(|\Delta\xi(s)|\,\chi_{\{s<\zeta_N\}})\,ds/\Im_{\tau_l}\Big)\\ &= CE\Big(\int_0^u q_N(|\Delta\xi(\eta_v)|\,\chi_{\{\eta_v<\zeta_N\}})\,dv/\Im_{\tau_l}\Big).\end{aligned}$$

Further we apply Jensen's inequality for concave functions and get

$$\begin{aligned}E(|\Delta\xi(\eta_u)|\chi_{\{\eta_u<\zeta_N\}}) &\leqslant C\int_0^u Eq_N(|\Delta\xi(\eta_v)|\,\chi_{\{\eta_v<\zeta_N\}})\,dv\\ &\leqslant Cq_N\Big(\int_0^u E(|\Delta\xi(\eta_v)|\,\chi_{\{\eta_v<\zeta_N\}})\,dv\Big).\end{aligned}$$

Thus, we have inequality

$$\int_0^u \alpha(v)q_N^{-1}\Big(\int_0^v \alpha(z)\,dz\Big)\,dv \leqslant Cu,$$

where $\alpha(u) = E(|\Delta\xi(\eta_u)|\chi_{\{\eta_u<\zeta_N\}})$. The last inequality together with condition (2) on the function $q_N(u)$ implies the equality $\alpha(u) = 0$ for all $u \geqslant 0$. Therefore $\xi(\eta_u) - \tilde{\xi}(\eta_u) = 0$ for $\tau_l \leqslant \eta_u < \tau_{l+1}$. Since $\xi(\eta_u) - \tilde{\xi}(\eta_u)$ has no discontinuity points of the second kind, we deduce that

$$\sup_{\tau_l\leqslant\eta_u<\tau_{l+1}} |\xi(\eta_u) - \tilde{\xi}(\eta_u)| = \sup_{\tau_l\leqslant s<\tau_{l+1}} |\xi(s) - \tilde{\xi}(s)| = 0.$$

Thus, $\xi(t) = \tilde{\xi}(t)$ for all $\tau_l \leqslant t < \tau_{l+1}$ and $\xi(\tau_{l+1}-) = \tilde{\xi}(\tau_{l+1}-)$. The proof will be complete if we show that $\xi(\tau_{l+1}) = \tilde{\xi}(\tau_{l+1})$. But it is indeed the case because it follows from the construction of the solution of equation (1) that

$$\begin{aligned}\xi(\tau_{l+1}) &= \xi(\tau_{l+1}-) + f_2(\tau_{l+1}, \xi(\tau_{l+1}-), \hat{\theta}_{l+1})\\ &= \tilde{\xi}(\tau_{l+1}-) + f_2(\tau_{l+1}, \tilde{\xi}(\tau_{l+1}-), \hat{\theta}_{l+1}) = \tilde{\xi}(\tau_{l+1}),\end{aligned}$$

where $\hat{\theta}_{l+1} \in \Theta_2$ is such that $\nu(\{\tau_{l+1}\}, \{\hat{\theta}_{l+1}\}) = 1$ (see also [2]: Theorem 1, §1, Chap. 4). □

Remark. If additionally to the assumptions of the theorem the following conditions hold true with probability 1:

(1) there exists a constant $C > 0$ such that

$$|a(t,x)|^2 + \sum_{k=1}^{m} |b_k(t,x)|^2 + \int_{\Theta_1} |f_1(t,x,\theta)|^2 \,\Pi(d\theta) \le C\big[1 + |x|^2\big];$$

(2) for arbitrary $t_1 > 0$, $x_1 \in \mathbb{R}^m$

$$\lim_{t\to t_1, x\to x_1} \int_{\Theta_1} |f_1(t,x,\theta) - f_1(t_1,x_1,\theta)|^2 \,\Pi(d\theta) = 0,$$

then equation (1) has a weak solution (see [2]: Theorem 3, §2, Chap. 4).

Corollary. *If the assumptions of the theorem and the conditions of the remark are satisfied, then equation (1) has a unique strong solution (see [2]: Theorem 9, §3, Chap. 6).*

3 Conclusion

In this work we investigate the problem of strong uniqueness of a solution for a system of stochastic differential equations with random coefficients and with differentials with respect to martingales, which may be continuous with probability 1 or discontinuous with jumps of Poisson type. Substantial generalizations of previously known results are got for the systems of the special type. For example, coefficients corresponding to the differentials of continuous martingales can be Hölder with index $\alpha \geqslant \frac{1}{2}$. Such systems are of particular interest, for instance, in connection with applications in finance for modeling instantaneous interest rate [1], in investigation of limiting behavior of unstable components of solutions of stochastic differential equations [3].

References

[1] Cox, J.C., Ingersoll, J.E., Ross, S.A.: A theory of the term structure of interest rate. Econometrica **53**, 385–407 (1985). MR0785475

[2] Gikhman, I.I., Skorokhod, A.V.: Stochastic Differential Equations and Their Applications. Naukova Dumka, Kiev (1982) (in Russian). MR0678374

[3] Kulinich, G.L.: Asymptotic behavior of unstable solutions of systems of stochastic diffusion equations. In: Proceedings of the Summer School on the Theory of Stochastic Processes, Vilnius, Part I, pp. 169–201 (1975). MR0501345

[4] Kulinich, G.L.: The existence and uniqueness of the solution of a stochastic differential equation with martingale differential. Theory Probab. Appl. **19**, 168–171 (1974). MR0345209

[5] Nakao, S.: On the pathwise uniqueness of solutions of one-dimensional stochastic differential equations. Osaka J. Math. **9**(3), 513–518 (1972). MR0326840

[6] Skorokhod, A.V.: On the existence and the uniqueness of solutions of stochastic diffusion equations. Sib. Math. J. **2**(1), 129–137 (1961). MR0132595

[7] Veretennikov, Yu, A.: On the strong solutions of stochastic differential equations. Teor. Veroyatn. Primen. **XXIV**(2), 348–360 (1979). MR0532447

[8] Yamada, T., Watanabe, S.: On the uniqueness of solutions of stochastic differential equations. J. Math. Kyoto Univ. **11**(1), 155–167 (1971). MR0278420

[9] Zubchenko, V.P., Mishura, Yu.S.: Existence and uniqueness of solutions of stochastic differential equations with non-Lipschitz diffusion and Poisson measure. Theory Probab. Math. Stat. **80**, 47–59 (2010). MR2541951